Λ_c polarimetry using the dominant hadronic mode — supplemental material

0.1.dev20+g837814f (24/08/2023 22:03:23)

Mikhail Mikhasenko, Remco de Boer, Miriam Fritsch

Aug 24, 2023

TABLE OF CONTENTS

DOI 10.48550/arXiv.2301.07010 DOI 10.5281/zenodo.7544989

Λ_c^+ polarimetry using the dominant hadronic mode The polarimeter vector field for multibody decays of a spin-half baryon is introduced as a generalisation of the baryon asymmetry parameters. Using a recent amplitude analysis of the $\Lambda_c^+ \to pK^-\pi^+$ decay performed at the LHCb experiment, we compute the distribution of the kinematic-dependent polarimeter vector for this process in the space of Mandelstam variables to express the polarised decay rate in a model-agnostic form. The obtained representation can facilitate polarisation measurements of the Λ_c^+ baryon and eases inclusion of the $\Lambda_c^+ \to pK^-\pi^+$ decay mode in hadronic amplitude analyses.

This website shows all analysis results that led to the publication of LHCb-PAPER-2022-044. More information on this publication can be found on the following pages:

- Publication on JHEP: J. High Energ. Phys. 2023, 228 (2023)
- Publication on arXiv: arXiv:2301.07010
- Record on CDS: cds.cern.ch/record/2838694
- Record for the source code on Zenodo: 10.5281/zenodo.7544989
- Archived documentation on GitLab Pages: lc2pkpi-polarimetry.docs.cern.ch
- Archived repository on CERN GitLab: gitlab.cern.ch/polarimetry/Lc2pKpi
- Active repository on GitHub containing discussions: github.com/ComPWA/polarimetry

---

Behind SSO login (LHCb members only)

- LHCb TWiki page: twiki.cern.ch/twiki/bin/viewauth/LHCbPhysics/PolarimetryLc2pKpi
- Charm WG meeting: indico.cern.ch/event/1187317
- RC approval presentation: indico.cern.ch/event/1213570
- Silent approval to submit: indico.cern.ch/event/1242323

Note: This document is a PDF rendering of the supplemental material hosted behind SSO-login on lc2pkpi-polarimetry.docs.cern.ch. Go to this webpage for a more extensive and interactive experience.

CHAPTER

ONE

NOMINAL AMPLITUDE MODEL

1.1 Resonances and LS-scheme

Particle definitions for Λ_c^+ and p, π^+, K^- in the sequential order.

name	LaTeX	J^P	mass (MeV)	width (MeV)
Lambda_c+	Λ_c^+	$\frac{1}{2}^+$	2,286	0
p	p	$\frac{1}{2}^+$	938	0
pi+	π^+	0^-	139	0
K-	K^-	0^-	493	0
Sigma-	Σ^-	$\frac{1}{2}^+$	1,189	0

Particle definitions as defined in `particle-definitions.yaml`:

name	LaTeX	J^P	mass (MeV)	width (MeV)
L(1405)	$\Lambda(1405)$	$\frac{1}{2}^-$	1,405	50
L(1520)	$\Lambda(1520)$	$\frac{3}{2}^-$	1,519	15
L(1600)	$\Lambda(1600)$	$\frac{1}{2}^+$	1,630	250
L(1670)	$\Lambda(1670)$	$\frac{1}{2}^-$	1,670	30
L(1690)	$\Lambda(1690)$	$\frac{3}{2}^-$	1,690	70
L(1800)	$\Lambda(1800)$	$\frac{1}{2}^-$	1,800	300
L(1810)	$\Lambda(1810)$	$\frac{1}{2}^+$	1,810	150
L(2000)	$\Lambda(2000)$	$\frac{1}{2}^-$	2,000	210
D(1232)	$\Delta(1232)$	$\frac{3}{2}^+$	1,232	117
D(1600)	$\Delta(1600)$	$\frac{3}{2}^+$	1,640	300
D(1620)	$\Delta(1620)$	$\frac{1}{2}^-$	1,620	130
D(1700)	$\Delta(1700)$	$\frac{3}{2}^-$	1,690	380
K(700)	$K(700)$	0^+	824	478
K(892)	$K(892)$	1^-	895	47
K(1410)	$K(1410)$	1^-	1,421	236
K(1430)	$K(1430)$	0^+	1,375	190

See also:

Amplitude model with LS-couplings (page 48)

Most models work take the **minimal L-value** in each LS-coupling (only model 17 works in the full LS-basis. The generated LS-couplings look as follows:

Only minimum LS (12)	All LS-couplings (26)
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+K^-$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+K^-$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+K^-$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(700) \xrightarrow[S=0]{L=0} \pi^+K^-p$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(700) \xrightarrow[S=0]{L=0} \pi^+K^-p$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} K(700) \xrightarrow[S=0]{L=0} \pi^+K^-p$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-p$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-p$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-p$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\pi^+$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$	$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$
	$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$
	$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$

Or with J^P-values:

Only minimum LS (12)	All LS-couplings (26)
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Delta(1232) \left[\frac{3}{2}^+\right] \xrightarrow[S=1/2]{L=1} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Delta(1232) \left[\frac{3}{2}^+\right] \xrightarrow[S=1/2]{L=1} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=2} \Delta(1232) \left[\frac{3}{2}^+\right] \xrightarrow[S=1/2]{L=1} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Delta(1600) \left[\frac{3}{2}^+\right] \xrightarrow[S=1/2]{L=1} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Delta(1600) \left[\frac{3}{2}^+\right] \xrightarrow[S=1/2]{L=1} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=2} \Delta(1600) \left[\frac{3}{2}^+\right] \xrightarrow[S=1/2]{L=1} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Delta(1700) \left[\frac{3}{2}^-\right] \xrightarrow[S=1/2]{L=2} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Delta(1700) \left[\frac{3}{2}^-\right] \xrightarrow[S=1/2]{L=2} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=2} \Delta(1700) \left[\frac{3}{2}^-\right] \xrightarrow[S=1/2]{L=2} p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right] K^- \left[0^-\right]$
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} K(700) \left[0^+\right] \xrightarrow[S=0]{L=0} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} K(700) \left[0^+\right] \xrightarrow[S=0]{L=0} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=1} K(700) \left[0^+\right] \xrightarrow[S=0]{L=0} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} K(892) \left[1^-\right] \xrightarrow[S=0]{L=1} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} K(892) \left[1^-\right] \xrightarrow[S=0]{L=1} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=1} K(892) \left[1^-\right] \xrightarrow[S=0]{L=1} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} K(892) \left[1^-\right] \xrightarrow[S=0]{L=1} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=2} K(892) \left[1^-\right] \xrightarrow[S=0]{L=1} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} K(1430) \left[0^+\right] \xrightarrow[S=0]{L=0} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} K(1430) \left[0^+\right] \xrightarrow[S=0]{L=0} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=1} K(1430) \left[0^+\right] \xrightarrow[S=0]{L=0} \pi^+ \left[0^-\right] K^- \left[0^-\right] p \left[\frac{1}{2}^+\right]$
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} \Lambda(1405) \left[\frac{1}{2}^-\right] \xrightarrow[S=1/2]{L=0} K^- \left[0^-\right] p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=0} \Lambda(1405) \left[\frac{1}{2}^-\right] \xrightarrow[S=1/2]{L=0} K^- \left[0^-\right] p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=1/2]{L=1} \Lambda(1405) \left[\frac{1}{2}^-\right] \xrightarrow[S=1/2]{L=0} K^- \left[0^-\right] p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right]$
$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Lambda(1520) \left[\frac{3}{2}^-\right] \xrightarrow[S=1/2]{L=2} K^- \left[0^-\right] p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right]$	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=1} \Lambda(1520) \left[\frac{3}{2}^-\right] \xrightarrow[S=1/2]{L=2} K^- \left[0^-\right] p \left[\frac{1}{2}^+\right] \pi^+ \left[0^-\right]$
	$\Lambda_c^+ \left[\frac{1}{2}^+\right] \xrightarrow[S=3/2]{L=2} \Lambda(1520) \left[\frac{3}{2}^-\right] \xrightarrow[S=1/2]{L=2}$

1.2 Amplitude

1.2.1 Spin-alignment amplitude

The full intensity of the amplitude model is obtained by summing the following aligned amplitude over all helicity values λ_i in the initial state 0 and final states $1, 2, 3$:

$$\sum_{\lambda_0^{\prime}=-1/2}^{1/2} \sum_{\lambda_1^{\prime}=-1/2}^{1/2} A^{1}_{\lambda_0^{\prime},\lambda_1^{\prime}} d^{\frac{1}{2}}_{\lambda_1^{\prime},\lambda_1}\left(\zeta^{1}_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0^{\prime}}\left(\zeta^{0}_{1(1)}\right) + A^{2}_{\lambda_0^{\prime},\lambda_1^{\prime}} d^{\frac{1}{2}}_{\lambda_1^{\prime},\lambda_1}\left(\zeta^{1}_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0^{\prime}}\left(\zeta^{0}_{2(1)}\right) + A^{3}_{\lambda_0^{\prime},\lambda_1^{\prime}} d^{\frac{1}{2}}_{\lambda_1^{\prime},\lambda_1}\left(\zeta^{1}_{3(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0^{\prime}}\left(\zeta^{0}_{3(1)}\right)$$

Note that we simplified notation here: the amplitude indices for the spinless states are not rendered and their corresponding Wigner-d alignment functions are simply 1.

The relevant $\zeta^i_{j(k)}$ angles are *defined as* (page 40):

$$\begin{aligned}
\zeta^{0}_{1(1)} &= 0\\
\zeta^{1}_{1(1)} &= 0\\
\zeta^{0}_{2(1)} &= -\operatorname{acos}\left(\frac{-2m_0^2(-m_1^2-m_2^2+\sigma_3)+(m_0^2+m_1^2-\sigma_1)(m_0^2+m_2^2-\sigma_2)}{\sqrt{\lambda(m_0^2,m_2^2,\sigma_2)}\sqrt{\lambda(m_0^2,\sigma_1,m_1^2)}}\right)\\
\zeta^{1}_{2(1)} &= \operatorname{acos}\left(\frac{2m_1^2(-m_0^2-m_3^2+\sigma_3)+(m_0^2+m_1^2-\sigma_1)(-m_1^2-m_3^2+\sigma_2)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(\sigma_2,m_1^2,m_3^2)}}\right)\\
\zeta^{0}_{3(1)} &= \operatorname{acos}\left(\frac{-2m_0^2(-m_1^2-m_3^2+\sigma_2)+(m_0^2+m_1^2-\sigma_1)(m_0^2+m_3^2-\sigma_3)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(m_0^2,\sigma_3,m_3^2)}}\right)\\
\zeta^{1}_{3(1)} &= -\operatorname{acos}\left(\frac{2m_1^2(-m_0^2-m_2^2+\sigma_2)+(m_0^2+m_1^2-\sigma_1)(-m_1^2-m_2^2+\sigma_3)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(\sigma_3,m_1^2,m_2^2)}}\right)
\end{aligned}$$

1.2.2 Sub-system amplitudes

$$\begin{aligned}
A^{1}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} -\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(892),0,0}\mathcal{H}^{\text{production}}_{K(892),\lambda_R,-\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} -\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}_{\text{Bugg}}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(1430),0,0}\\
A^{2}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} -\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1520),0,-\frac{1}{2}}\mathcal{H}^{\text{production}}_{L(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} -\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1600),0,-\frac{1}{2}}\mathcal{H}\\
A^{3}_{-\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1232),-\frac{1}{2},0}\mathcal{H}^{\text{production}}_{D(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1600),-\frac{1}{2},0}\mathcal{H}\\
A^{1}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} \delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(892),0,0}\mathcal{H}^{\text{production}}_{K(892),\lambda_R,\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} \delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}_{\text{Bugg}}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(1430),0,0}\mathcal{H}\\
A^{2}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1520),0,\frac{1}{2}}\mathcal{H}^{\text{production}}_{L(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} \delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1600),0,\frac{1}{2}}\mathcal{H}\\
A^{3}_{-\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1232),\frac{1}{2},0}\mathcal{H}^{\text{production}}_{D(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{-\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1600),\frac{1}{2},0}\mathcal{H}\\
A^{1}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} -\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(892),0,0}\mathcal{H}^{\text{production}}_{K(892),\lambda_R,-\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} -\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}_{\text{Bugg}}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(1430),0,0}\mathcal{H}\\
A^{2}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} -\delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1520),0,-\frac{1}{2}}\mathcal{H}^{\text{production}}_{L(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} -\delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1600),0,-\frac{1}{2}}\mathcal{H}\\
A^{3}_{\frac{1}{2},-\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1232),-\frac{1}{2},0}\mathcal{H}^{\text{production}}_{D(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1600),-\frac{1}{2},0}\mathcal{H}\\
A^{1}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-1}^{1} \delta_{\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(892),0,0}\mathcal{H}^{\text{production}}_{K(892),\lambda_R,\frac{1}{2}} d^{1}_{\lambda_R,0}\left(\theta_{23}\right) + \sum_{\lambda_R=0} \delta_{\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}_{\text{Bugg}}\left(\sigma_1\right)\mathcal{H}^{\text{decay}}_{K(1430),0,0}\mathcal{H}\\
A^{2}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1520),0,\frac{1}{2}}\mathcal{H}^{\text{production}}_{L(1520),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}\left(\theta_{31}\right) + \sum_{\lambda_R=-1/2}^{1/2} \delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_2\right)\mathcal{H}^{\text{decay}}_{L(1600),0,\frac{1}{2}}\mathcal{H}\\
A^{3}_{\frac{1}{2},\frac{1}{2}} &= \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1232),\frac{1}{2},0}\mathcal{H}^{\text{production}}_{D(1232),\lambda_R,0} d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}\left(\theta_{12}\right) + \sum_{\lambda_R=-3/2}^{3/2} \delta_{\frac{1}{2}\lambda_R}\mathcal{R}\left(\sigma_3\right)\mathcal{H}^{\text{decay}}_{D(1600),\frac{1}{2},0}\mathcal{H}
\end{aligned}$$

The θ_{ij} angles are *defined as* (page 40):

$$
\begin{aligned}
\theta_{23} &= \operatorname{acos}\left(\frac{2\sigma_1(-m_1^2-m_2^2+\sigma_3)-(m_0^2-m_1^2-\sigma_1)(m_2^2-m_3^2+\sigma_1)}{\sqrt{\lambda(m_0^2,m_1^2,\sigma_1)}\sqrt{\lambda(\sigma_1,m_2^2,m_3^2)}}\right) \\
\theta_{31} &= \operatorname{acos}\left(\frac{2\sigma_2(-m_2^2-m_3^2+\sigma_1)-(m_0^2-m_2^2-\sigma_2)(-m_1^2+m_3^2+\sigma_2)}{\sqrt{\lambda(m_0^2,m_2^2,\sigma_2)}\sqrt{\lambda(\sigma_2,m_3^2,m_1^2)}}\right) \\
\theta_{12} &= \operatorname{acos}\left(\frac{2\sigma_3(-m_1^2-m_3^2+\sigma_2)-(m_0^2-m_3^2-\sigma_3)(m_1^2-m_2^2+\sigma_3)}{\sqrt{\lambda(m_0^2,m_3^2,\sigma_3)}\sqrt{\lambda(\sigma_3,m_1^2,m_2^2)}}\right)
\end{aligned}
$$

Definitions for the ϕ_{ij} angles can be found under *DPD angles* (page 40).

1.3 Parameter definitions

Parameter values are provided in `model-definitions.yaml`, but the **keys** of the helicity couplings have to remapped to the helicity **symbols** that are used in this amplitude model. The function `parameter_key_to_symbol()` (page 63) implements this remapping, following the supplementary material of [1]. It is asserted below that:

1. the keys are mapped to symbols that exist in the nominal amplitude model
2. all parameter symbols in the nominal amplitude model have a value assigned to them.

1.3.1 Helicity coupling values

Production couplings

$$
\begin{aligned}
\mathcal{H}^\text{production}_{K(892),-1,-\frac{1}{2}} &= 1.192614-1.025814i \\
\mathcal{H}^\text{production}_{L(1405),-\frac{1}{2},0} &= -4.572486+3.190144i \\
\mathcal{H}^\text{production}_{L(1520),-\frac{1}{2},0} &= 0.293998+0.044324i \\
\mathcal{H}^\text{production}_{L(1600),-\frac{1}{2},0} &= -4.840649-3.082786i \\
\mathcal{H}^\text{production}_{L(1670),-\frac{1}{2},0} &= -0.339585-0.144678i \\
\mathcal{H}^\text{production}_{L(1690),-\frac{1}{2},0} &= -0.385772-0.110235i \\
\mathcal{H}^\text{production}_{L(2000),-\frac{1}{2},0} &= -8.014857-7.614006i \\
\mathcal{H}^\text{production}_{D(1232),-\frac{1}{2},0} &= -6.778191+3.051805i \\
\mathcal{H}^\text{production}_{D(1600),-\frac{1}{2},0} &= 11.401585-3.125511i \\
\mathcal{H}^\text{production}_{D(1700),-\frac{1}{2},0} &= -10.37828-1.434872i \\
\mathcal{H}^\text{production}_{K(700),0,\frac{1}{2}} &= 0.068908+2.521444i \\
\mathcal{H}^\text{production}_{K(892),0,\frac{1}{2}} &= -0.727145-4.155027i \\
\mathcal{H}^\text{production}_{K(1430),0,\frac{1}{2}} &= -6.71516+10.479411i \\
\mathcal{H}^\text{production}_{K(700),0,-\frac{1}{2}} &= -2.68563+0.03849i \\
\mathcal{H}^\text{production}_{K(892),0,-\frac{1}{2}} &= 1+0i \\
\mathcal{H}^\text{production}_{K(1430),0,-\frac{1}{2}} &= 0.219754+8.741196i \\
\mathcal{H}^\text{production}_{L(1405),\frac{1}{2},0} &= 10.44608+2.787441i \\
\mathcal{H}^\text{production}_{L(1520),\frac{1}{2},0} &= -0.160687+1.498833i \\
\mathcal{H}^\text{production}_{L(1600),\frac{1}{2},0} &= 6.971233-0.842435i \\
\mathcal{H}^\text{production}_{L(1670),\frac{1}{2},0} &= -0.570978+1.011833i \\
\mathcal{H}^\text{production}_{L(1690),\frac{1}{2},0} &= -2.730592-0.353613i \\
\mathcal{H}^\text{production}_{L(2000),\frac{1}{2},0} &= -4.336255-3.796192i \\
\mathcal{H}^\text{production}_{D(1232),\frac{1}{2},0} &= -12.987193+4.528336i \\
\mathcal{H}^\text{production}_{D(1600),\frac{1}{2},0} &= 6.729211-1.002383i \\
\mathcal{H}^\text{production}_{D(1700),\frac{1}{2},0} &= -12.874102-2.10557i \\
\mathcal{H}^\text{production}_{K(892),1,\frac{1}{2}} &= -3.141446-3.29341i
\end{aligned}
$$

Decay couplings

$$
\begin{aligned}
\mathcal{H}^\text{decay}_{K(892),0,0} &= 1 \\
\mathcal{H}^\text{decay}_{L(1405),0,-\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{L(1520),0,-\frac{1}{2}} &= -1 \\
\mathcal{H}^\text{decay}_{L(1600),0,-\frac{1}{2}} &= -1 \\
\mathcal{H}^\text{decay}_{L(1670),0,-\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{L(1690),0,-\frac{1}{2}} &= -1 \\
\mathcal{H}^\text{decay}_{L(2000),0,-\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{D(1232),-\frac{1}{2},0} &= 1 \\
\mathcal{H}^\text{decay}_{D(1600),-\frac{1}{2},0} &= 1 \\
\mathcal{H}^\text{decay}_{D(1700),-\frac{1}{2},0} &= -1 \\
\mathcal{H}^\text{decay}_{K(700),0,0} &= 1 \\
\mathcal{H}^\text{decay}_{K(1430),0,0} &= 1 \\
\mathcal{H}^\text{decay}_{L(1405),0,\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{L(1520),0,\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{L(1600),0,\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{L(1670),0,\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{L(1690),0,\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{L(2000),0,\frac{1}{2}} &= 1 \\
\mathcal{H}^\text{decay}_{D(1232),\frac{1}{2},0} &= 1 \\
\mathcal{H}^\text{decay}_{D(1600),\frac{1}{2},0} &= 1 \\
\mathcal{H}^\text{decay}_{D(1700),\frac{1}{2},0} &= 1
\end{aligned}
$$

1.3.2 Non-coupling parameters

$$
\begin{aligned}
R_{\mathrm{res}} &= 1.5 \\
R_{\Lambda_c} &= 5 \\
\Gamma_{D(1232)} &= 0.117 \\
\Gamma_{D(1600)} &= 0.3 \\
\Gamma_{D(1700)} &= 0.38 \\
\Gamma_{K(1430)} &= 0.19 \\
\Gamma_{K(700)} &= 0.47800000000000004 \\
\Gamma_{K(892)} &= 0.047299999999999995 \\
\Gamma_{L(1405) \to \Sigma^- \pi^+} &= 0.0505 \\
\Gamma_{L(1405) \to p K^-} &= 0.0505 \\
\Gamma_{L(1520)} &= 0.015195 \\
\Gamma_{L(1600)} &= 0.25 \\
\Gamma_{L(1670)} &= 0.03 \\
\Gamma_{L(1690)} &= 0.07 \\
\Gamma_{L(2000)} &= 0.17926 \\
\gamma_{K(1430)} &= 0.020981 \\
\gamma_{K(700)} &= 0.94106 \\
m_0 &= 2.28646 \\
m_1 &= 0.938272046 \\
m_2 &= 0.13957018 \\
m_3 &= 0.49367700000000003 \\
m_{D(1232)} &= 1.232 \\
m_{D(1600)} &= 1.6400000000000001 \\
m_{D(1700)} &= 1.69 \\
m_{K(1430)} &= 1.375 \\
m_{K(700)} &= 0.8240000000000001 \\
m_{K(892)} &= 0.8955000000000001 \\
m_{K-} &= 0.49367700000000003 \\
m_{L(1405)} &= 1.4051 \\
m_{L(1520)} &= 1.518467 \\
m_{L(1600)} &= 1.6300000000000001 \\
m_{L(1670)} &= 1.67 \\
m_{L(1690)} &= 1.69 \\
m_{L(2000)} &= 1.98819 \\
m_{Lambda_c+} &= 2.28646 \\
m_{Sigma-} &= 1.1893699999999998 \\
m_{pi+} &= 0.13957018 \\
m_p &= 0.938272046
\end{aligned}
$$

CHAPTER

TWO

CROSS-CHECK WITH LHCB DATA

2.1 Lineshape comparison

We compute a few lineshapes for the following point in phase space and compare it with the values from [1]:

```
{'costhetap': -0.9949949110827053,
 'm2kpi': 0.7980703453578917,
 'm2pk': 3.6486261122281745,
 'phikpi': -0.4,
 'phip': -0.3}
```

The lineshapes are computed for the following decay chains:

$$\begin{array}{c}
\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(892) \xrightarrow[S=0]{L=1} \pi^+ K^- p \\
\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1405) \xrightarrow[S=1/2]{L=0} K^- p \pi^+ \\
\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Lambda(1690) \xrightarrow[S=1/2]{L=2} K^- p \pi^+
\end{array}$$

```
{'BW_K(892)_p^1_q^0': '(2.1687201455088894+23.58225917009096j)',
 'BW_L(1405)_p^0_q^0': '(-0.5636481410171861+0.13763637759224928j)',
 'BW_L(1690)_p^2_q^1': '(-1.5078327158518026+0.9775036395061584j)'}
```

$$\begin{array}{c}
2.16872014550901 + 23.5822591700909i \\
-0.563648141017186 + 0.137636377592249i \\
-1.5078327158518 + 0.977503639506157i
\end{array}$$

Tip: These values are **equal up to 13 decimals**.

2.2 Amplitude comparison

The amplitude for each decay chain and each outer state helicity combination are evaluated on the following point in phase space:

$$
\begin{aligned}
\theta_{23} &= 1.821341166520149 \\
\theta_{31} &= 1.8038351483715633 \\
\theta_{12} &= 1.1139045236042229 \\
\zeta^0_{1(1)} &= 0.0 \\
\zeta^1_{1(1)} &= 0.0 \\
\zeta^0_{2(1)} &= -2.0777687076712614 \\
\zeta^1_{2(1)} &= 0.22583331080386268 \\
\zeta^0_{3(1)} &= 2.6540796539955838 \\
\zeta^1_{3(1)} &= -0.5594175047790548 \\
\sigma_1 &= 0.7980703453578917 \\
\sigma_2 &= 3.6486261122281745 \\
\sigma_3 &= 1.9247541217931925
\end{aligned}
$$

2.2.1 Default model

Tip: Computed amplitudes are equal to LHCb amplitudes up to **13 decimals**.

	Computed	Expected	Difference
ArD(1232)1	$\mathcal{H}^{\text{production}}_{D(1232),-\frac{1}{2},0}$		
A++	-0.488498+0.517710j	-0.488498+0.517710j	3.11e-14
A+-	0.894898-0.948412j	0.894898-0.948412j	7.61e-15
A-+	0.121490-0.128755j	0.121490-0.128755j	1.80e-14
A--	-0.222563+0.235872j	-0.222563+0.235872j	6.14e-15
ArD(1232)2	$\mathcal{H}^{\text{production}}_{D(1232),\frac{1}{2},0}$		
A++	-0.222563+0.235872j	-0.222563+0.235872j	6.14e-15
A+-	-0.121490+0.128755j	-0.121490+0.128755j	1.80e-14
A-+	-0.894898+0.948412j	-0.894898+0.948412j	7.61e-15
A--	-0.488498+0.517710j	-0.488498+0.517710j	3.11e-14
ArD(1600)1	$\mathcal{H}^{\text{production}}_{D(1600),-\frac{1}{2},0}$		
A++	0.289160+0.081910j	0.289160+0.081910j	3.11e-14
A+-	-0.529724-0.150054j	-0.529724-0.150054j	7.48e-15
A-+	-0.071915-0.020371j	-0.071915-0.020371j	1.82e-14
A--	0.131743+0.037319j	0.131743+0.037319j	5.71e-15
ArD(1600)2	$\mathcal{H}^{\text{production}}_{D(1600),\frac{1}{2},0}$		
A++	0.131743+0.037319j	0.131743+0.037319j	5.71e-15
A+-	0.071915+0.020371j	0.071915+0.020371j	1.82e-14
A-+	0.529724+0.150054j	0.529724+0.150054j	7.48e-15
A--	0.289160+0.081910j	0.289160+0.081910j	3.11e-14
ArD(1700)1	$\mathcal{H}^{\text{production}}_{D(1700),-\frac{1}{2},0}$		
A++	-0.018885-0.001757j	-0.018885-0.001757j	3.18e-13
A+-	0.315695+0.029366j	0.315695+0.029366j	2.04e-14
A-+	0.004697+0.000437j	0.004697+0.000437j	3.30e-13
A--	-0.078514-0.007303j	-0.078514-0.007303j	7.22e-15
ArD(1700)2	$\mathcal{H}^{\text{production}}_{D(1700),\frac{1}{2},0}$		
A++	0.078514+0.007303j	0.078514+0.007303j	7.22e-15
A+-	0.004697+0.000437j	0.004697+0.000437j	3.30e-13
A-+	0.315695+0.029366j	0.315695+0.029366j	2.04e-14

continues on next page

Table 2.1 – continued from previous page

	Computed	Expected	Difference
`A--`	0.018885+0.001757j	0.018885+0.001757j	3.18e-13
`ArK(892)1`	$\mathcal{H}^\text{production}_{K(892),0,-\frac{1}{2}}$		
`A++`	-0.537695-5.846793j	-0.537695-5.846793j	5.00e-15
`A+-`	0.000000+0.000000j	0.000000+0.000000j	
`A-+`	-0.000000+0.000000j	0.000000+0.000000j	
`A--`	0.000000+0.000000j	0.000000+0.000000j	
`ArK(892)2`	$\mathcal{H}^\text{production}_{K(892),-1,-\frac{1}{2}}$		
`A++`	-0.000000+0.000000j	0.000000+0.000000j	
`A+-`	0.000000+0.000000j	0.000000+0.000000j	
`A-+`	1.485636+16.154534j	1.485636+16.154534j	3.20e-15
`A--`	0.000000+0.000000j	0.000000+0.000000j	
`ArK(892)3`	$\mathcal{H}^\text{production}_{K(892),1,\frac{1}{2}}$		
`A++`	-0.000000+0.000000j	0.000000+0.000000j	
`A+-`	-1.485636-16.154534j	-1.485636-16.154534j	3.20e-15
`A-+`	-0.000000+0.000000j	0.000000+0.000000j	
`A--`	0.000000+0.000000j	0.000000+0.000000j	
`ArK(892)4`	$\mathcal{H}^\text{production}_{K(892),0,\frac{1}{2}}$		
`A++`	-0.000000+0.000000j	0.000000+0.000000j	
`A+-`	0.000000+0.000000j	0.000000+0.000000j	
`A-+`	-0.000000+0.000000j	0.000000+0.000000j	
`A--`	-0.537695-5.846793j	-0.537695-5.846793j	5.00e-15
`ArK(1430)1`	$\mathcal{H}^\text{production}_{K(1430),0,\frac{1}{2}}$		
`A++`	-0.000000+0.000000j	0.000000+0.000000j	
`A+-`	0.000000+0.000000j	0.000000+0.000000j	
`A-+`	-0.000000+0.000000j	0.000000+0.000000j	
`A--`	0.909456+0.072819j	0.909456+0.072819j	1.22e-16
`ArK(1430)2`	$\mathcal{H}^\text{production}_{K(1430),0,-\frac{1}{2}}$		
`A++`	0.909456+0.072819j	0.909456+0.072819j	1.22e-16
`A+-`	0.000000+0.000000j	0.000000+0.000000j	
`A-+`	-0.000000+0.000000j	0.000000+0.000000j	
`A--`	0.000000+0.000000j	0.000000+0.000000j	
`ArK(700)1`	$\mathcal{H}^\text{production}_{K(700),0,\frac{1}{2}}$		
`A++`	-0.000000+0.000000j	0.000000+0.000000j	
`A+-`	0.000000+0.000000j	0.000000+0.000000j	
`A-+`	-0.000000+0.000000j	0.000000+0.000000j	
`A--`	-1.708879+3.380634j	-1.708879+3.380634j	4.97e-16
`ArK(700)2`	$\mathcal{H}^\text{production}_{K(700),0,-\frac{1}{2}}$		
`A++`	-1.708879+3.380634j	-1.708879+3.380634j	4.97e-16
`A+-`	0.000000+0.000000j	0.000000+0.000000j	
`A-+`	-0.000000+0.000000j	0.000000+0.000000j	
`A--`	0.000000+0.000000j	0.000000+0.000000j	
`ArL(1405)1`	$\mathcal{H}^\text{production}_{L(1405),-\frac{1}{2},0}$		
`A++`	-0.412613+0.100755j	-0.412613+0.100755j	1.49e-15
`A+-`	-0.256372+0.062603j	-0.256372+0.062603j	3.06e-15
`A-+`	-0.242818+0.059293j	-0.242818+0.059293j	1.40e-15
`A--`	-0.150872+0.036841j	-0.150872+0.036841j	3.06e-15
`ArL(1405)2`	$\mathcal{H}^\text{production}_{L(1405),\frac{1}{2},0}$		
`A++`	-0.150872+0.036841j	-0.150872+0.036841j	3.06e-15
`A+-`	0.242818-0.059293j	0.242818-0.059293j	1.40e-15
`A-+`	0.256372-0.062603j	0.256372-0.062603j	3.06e-15

continues on next page

Table 2.1 – continued from previous page

	Computed	Expected	Difference
`A--`	-0.412613+0.100755j	-0.412613+0.100755j	1.49e-15
`ArL(1520)1`	$\mathcal{H}^\text{production}_{L(1520),-\frac{1}{2},0}$		
`A++`	0.257632-0.288056j	0.257632-0.288056j	1.52e-14
`A+-`	0.731594-0.817988j	0.731594-0.817988j	2.23e-14
`A-+`	0.151613-0.169517j	0.151613-0.169517j	1.51e-14
`A--`	0.430534-0.481376j	0.430534-0.481376j	2.22e-14
`ArL(1520)2`	$\mathcal{H}^\text{production}_{L(1520),\frac{1}{2},0}$		
`A++`	-0.430534+0.481376j	-0.430534+0.481376j	2.22e-14
`A+-`	0.151613-0.169517j	0.151613-0.169517j	1.51e-14
`A-+`	0.731594-0.817988j	0.731594-0.817988j	2.25e-14
`A--`	-0.257632+0.288056j	-0.257632+0.288056j	1.52e-14
`ArL(1600)1`	$\mathcal{H}^\text{production}_{L(1600),-\frac{1}{2},0}$		
`A++`	-0.385436+0.424707j	-0.385436+0.424707j	1.17e-15
`A+-`	0.382669-0.421658j	0.382669-0.421658j	3.88e-15
`A-+`	-0.226825+0.249935j	-0.226825+0.249935j	1.38e-15
`A--`	0.225196-0.248141j	0.225196-0.248141j	3.64e-15
`ArL(1600)2`	$\mathcal{H}^\text{production}_{L(1600),\frac{1}{2},0}$		
`A++`	-0.225196+0.248141j	-0.225196+0.248141j	3.68e-15
`A+-`	-0.226825+0.249935j	-0.226825+0.249935j	1.46e-15
`A-+`	0.382669-0.421658j	0.382669-0.421658j	3.88e-15
`A--`	0.385436-0.424707j	0.385436-0.424707j	1.17e-15
`ArL(1670)1`	$\mathcal{H}^\text{production}_{L(1670),-\frac{1}{2},0}$		
`A++`	-0.846639+0.064025j	-0.846639+0.064025j	1.18e-15
`A+-`	-0.526049+0.039781j	-0.526049+0.039781j	2.96e-15
`A-+`	-0.498237+0.037678j	-0.498237+0.037678j	1.22e-15
`A--`	-0.309574+0.023411j	-0.309574+0.023411j	3.24e-15
`ArL(1670)2`	$\mathcal{H}^\text{production}_{L(1670),\frac{1}{2},0}$		
`A++`	-0.309574+0.023411j	-0.309574+0.023411j	3.24e-15
`A+-`	0.498237-0.037678j	0.498237-0.037678j	1.22e-15
`A-+`	0.526049-0.039781j	0.526049-0.039781j	2.96e-15
`A--`	-0.846639+0.064025j	-0.846639+0.064025j	1.18e-15
`ArL(1690)1`	$\mathcal{H}^\text{production}_{L(1690),-\frac{1}{2},0}$		
`A++`	0.232446-0.150691j	0.232446-0.150691j	1.63e-14
`A+-`	0.660073-0.427915j	0.660073-0.427915j	2.30e-14
`A-+`	0.136791-0.088680j	0.136791-0.088680j	1.62e-14
`A--`	0.388445-0.251823j	0.388445-0.251823j	2.29e-14
`ArL(1690)2`	$\mathcal{H}^\text{production}_{L(1690),\frac{1}{2},0}$		
`A++`	-0.388445+0.251823j	-0.388445+0.251823j	2.31e-14
`A+-`	0.136791-0.088680j	0.136791-0.088680j	1.62e-14
`A-+`	0.660073-0.427915j	0.660073-0.427915j	2.32e-14
`A--`	-0.232446+0.150691j	-0.232446+0.150691j	1.63e-14
`ArL(2000)1`	$\mathcal{H}^\text{production}_{L(2000),-\frac{1}{2},0}$		
`A++`	1.072514+1.195841j	1.072514+1.195841j	1.47e-15
`A+-`	0.666394+0.743022j	0.666394+0.743022j	2.69e-15
`A-+`	0.631162+0.703738j	0.631162+0.703738j	1.34e-15
`A--`	0.392165+0.437260j	0.392165+0.437260j	3.03e-15
`ArL(2000)2`	$\mathcal{H}^\text{production}_{L(2000),\frac{1}{2},0}$		
`A++`	0.392165+0.437260j	0.392165+0.437260j	3.03e-15
`A+-`	-0.631162-0.703738j	-0.631162-0.703738j	1.34e-15
`A-+`	-0.666394-0.743022j	-0.666394-0.743022j	2.69e-15

continues on next page

Table 2.1 – continued from previous page

	Computed	Expected	Difference
A--	1.072514+1.195841j	1.072514+1.195841j	1.47e-15

2.2.2 LS-model

Tip: Computed amplitudes are equal to LHCb amplitudes up to **13 decimals**.

	Computed	Expected	Difference
ArD(1232)1	$\mathcal{H}^{\mathrm{LS,production}}_{D(1232),1,\frac{3}{2}}$		
A++	0.502796-0.532862j	0.502796-0.532862j	1.94e-14
A+-	-0.546882+0.579585j	-0.546882+0.579585j	5.67e-15
A-+	0.546882-0.579585j	0.546882-0.579585j	5.67e-15
A--	0.502796-0.532862j	0.502796-0.532862j	1.93e-14
ArD(1232)2	$\mathcal{H}^{\mathrm{LS,production}}_{D(1232),2,\frac{3}{2}}$		
A++	-0.180489+0.191282j	-0.180489+0.191282j	5.53e-14
A+-	0.689818-0.731068j	0.689818-0.731068j	2.67e-15
A-+	0.689818-0.731068j	0.689818-0.731068j	2.56e-15
A--	0.180489-0.191282j	0.180489-0.191282j	5.51e-14
ArD(1600)1	$\mathcal{H}^{\mathrm{LS,production}}_{D(1600),1,\frac{3}{2}}$		
A++	-0.297624-0.084307j	-0.297624-0.084307j	1.83e-14
A+-	0.323720+0.091699j	0.323720+0.091699j	4.47e-15
A-+	-0.323720-0.091699j	-0.323720-0.091699j	4.47e-15
A--	-0.297624-0.084307j	-0.297624-0.084307j	1.83e-14
ArD(1600)2	$\mathcal{H}^{\mathrm{LS,production}}_{D(1600),2,\frac{3}{2}}$		
A++	0.143541+0.040660j	0.143541+0.040660j	5.53e-14
A+-	-0.548604-0.155402j	-0.548604-0.155402j	2.35e-15
A-+	-0.548604-0.155402j	-0.548604-0.155402j	2.20e-15
A--	-0.143541-0.040660j	-0.143541-0.040660j	5.47e-14
ArD(1700)1	$\mathcal{H}^{\mathrm{LS,production}}_{D(1700),1,\frac{3}{2}}$		
A++	-0.042164-0.003922j	-0.042164-0.003922j	1.10e-13
A+-	-0.226551-0.021074j	-0.226551-0.021074j	1.47e-14
A-+	-0.226551-0.021074j	-0.226551-0.021074j	1.47e-14
A--	0.042164+0.003922j	0.042164+0.003922j	1.11e-13
ArD(1700)2	$\mathcal{H}^{\mathrm{LS,production}}_{D(1700),2,\frac{3}{2}}$		
A++	-0.105349-0.009800j	-0.105349-0.009800j	5.81e-14
A+-	0.336381+0.031290j	0.336381+0.031290j	2.34e-14
A-+	-0.336381-0.031290j	-0.336381-0.031290j	2.34e-14
A--	-0.105349-0.009800j	-0.105349-0.009800j	5.81e-14
ArK(892)1	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),0,\frac{1}{2}}$		
A++	0.219513+2.386943j	0.219513+2.386943j	5.19e-15
A+-	-0.857733-9.326825j	-0.857733-9.326825j	3.53e-15
A-+	-0.857733-9.326825j	-0.857733-9.326825j	3.53e-15
A--	-0.219513-2.386943j	-0.219513-2.386943j	5.19e-15
ArK(892)2	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),1,\frac{1}{2}}$		
A++	0.219549+2.387337j	0.219549+2.387337j	7.53e-15
A+-	-0.857874-9.328364j	-0.857874-9.328364j	2.80e-15
A-+	0.857874+9.328364j	0.857874+9.328364j	2.80e-15
A--	0.219549+2.387337j	0.219549+2.387337j	7.53e-15

continues on next page

Table 2.2 – continued from previous page

	Computed	Expected	Difference
ArK(892)3	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),1,\frac{3}{2}}$		
A++	0.310489+3.376204j	0.310489+3.376204j	4.72e-15
A+-	0.606609+6.596150j	0.606609+6.596150j	2.80e-15
A-+	-0.606609-6.596150j	-0.606609-6.596150j	2.80e-15
A--	0.310489+3.376204j	0.310489+3.376204j	4.72e-15
ArK(892)4	$\mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}}$		
A++	0.310629+3.377724j	0.310629+3.377724j	1.42e-14
A+-	0.606882+6.599119j	0.606882+6.599119j	7.92e-15
A-+	0.606882+6.599119j	0.606882+6.599119j	7.92e-15
A--	-0.310629-3.377724j	-0.310629-3.377724j	1.42e-14
ArK(1430)1	$\mathcal{H}^{\mathrm{LS,production}}_{K(1430),0,\frac{1}{2}}$		
A++	0.643091+0.051436j	0.643091+0.051436j	1.29e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.643091+0.051436j	0.643091+0.051436j	1.29e-16
ArK(1430)2	$\mathcal{H}^{\mathrm{LS,production}}_{K(1430),1,\frac{1}{2}}$		
A++	-0.643091-0.051436j	-0.643091-0.051436j	2.22e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	0.643091+0.051436j	0.643091+0.051436j	2.22e-16
ArK(700)1	$\mathcal{H}^{\mathrm{LS,production}}_{K(700),0,\frac{1}{2}}$		
A++	-1.070937+2.282902j	-1.070937+2.282902j	3.94e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	-1.070937+2.282902j	-1.070937+2.282902j	3.94e-16
ArK(700)2	$\mathcal{H}^{\mathrm{LS,production}}_{K(700),1,\frac{1}{2}}$		
A++	1.070937-2.282902j	1.070937-2.282902j	4.40e-16
A+-	0.000000+0.000000j	0.000000+0.000000j	
A-+	-0.000000+0.000000j	0.000000+0.000000j	
A--	-1.070937+2.282902j	-1.070937+2.282902j	4.40e-16
ArL(1405)1	$\mathcal{H}^{\mathrm{LS,production}}_{L(1405),0,\frac{1}{2}}$		
A++	-0.398444+0.097295j	-0.398444+0.097295j	8.48e-16
A+-	-0.009584+0.002340j	-0.009584+0.002340j	7.95e-14
A-+	0.009584-0.002340j	0.009584-0.002340j	8.06e-14
A--	-0.398444+0.097295j	-0.398444+0.097295j	8.48e-16
ArL(1405)2	$\mathcal{H}^{\mathrm{LS,production}}_{L(1405),1,\frac{1}{2}}$		
A++	0.163270-0.039869j	0.163270-0.039869j	2.06e-14
A+-	0.311387-0.076037j	0.311387-0.076037j	2.48e-14
A-+	0.311387-0.076037j	0.311387-0.076037j	2.50e-14
A--	-0.163270+0.039869j	-0.163270+0.039869j	2.06e-14
ArL(1520)1	$\mathcal{H}^{\mathrm{LS,production}}_{L(1520),1,\frac{3}{2}}$		
A++	0.117387-0.135999j	0.117387-0.135999j	3.04e-14
A+-	-0.599627+0.694701j	-0.599627+0.694701j	1.89e-14
A-+	-0.599627+0.694701j	-0.599627+0.694701j	1.90e-14
A--	-0.117387+0.135999j	-0.117387+0.135999j	3.03e-14
ArL(1520)2	$\mathcal{H}^{\mathrm{LS,production}}_{L(1520),2,\frac{3}{2}}$		
A++	0.330006-0.382330j	0.330006-0.382330j	7.41e-14
A+-	0.278127-0.322225j	0.278127-0.322225j	7.88e-14
A-+	-0.278127+0.322225j	-0.278127+0.322225j	7.87e-14
A--	0.330006-0.382330j	0.330006-0.382330j	7.41e-14

continues on next page

Table 2.2 – continued from previous page

	Computed	Expected	Difference
ArL(1600)1	$\mathcal{H}^{\text{LS,production}}_{L(1600),0,\frac{1}{2}}$		
A++	-0.431782+0.475775j	-0.431782+0.475775j	1.51e-15
A+-	0.110199-0.121426j	0.110199-0.121426j	9.38e-15
A-+	0.110199-0.121426j	0.110199-0.121426j	9.90e-15
A--	0.431782-0.475775j	0.431782-0.475775j	1.42e-15
ArL(1600)2	$\mathcal{H}^{\text{LS,production}}_{L(1600),1,\frac{1}{2}}$		
A++	0.102310-0.112734j	0.102310-0.112734j	3.06e-14
A+-	-0.389148+0.428797j	-0.389148+0.428797j	2.20e-14
A-+	0.389148-0.428797j	0.389148-0.428797j	2.21e-14
A--	0.102310-0.112734j	0.102310-0.112734j	3.06e-14
ArL(1670)1	$\mathcal{H}^{\text{LS,production}}_{L(1670),0,\frac{1}{2}}$		
A++	-0.817566+0.061827j	-0.817566+0.061827j	1.60e-16
A+-	-0.019666+0.001487j	-0.019666+0.001487j	7.55e-14
A-+	0.019666-0.001487j	0.019666-0.001487j	7.62e-14
A--	-0.817566+0.061827j	-0.817566+0.061827j	1.60e-16
ArL(1670)2	$\mathcal{H}^{\text{LS,production}}_{L(1670),1,\frac{1}{2}}$		
A++	0.345271-0.026110j	0.345271-0.026110j	1.85e-14
A+-	0.658498-0.049798j	0.658498-0.049798j	2.38e-14
A-+	0.658498-0.049798j	0.658498-0.049798j	2.36e-14
A--	-0.345271+0.026110j	-0.345271+0.026110j	1.87e-14
ArL(1690)1	$\mathcal{H}^{\text{LS,production}}_{L(1690),1,\frac{3}{2}}$		
A++	0.110308-0.071511j	0.110308-0.071511j	2.95e-14
A+-	-0.563468+0.365287j	-0.563468+0.365287j	1.82e-14
A-+	-0.563468+0.365287j	-0.563468+0.365287j	1.80e-14
A--	-0.110308+0.071511j	-0.110308+0.071511j	2.97e-14
ArL(1690)2	$\mathcal{H}^{\text{LS,production}}_{L(1690),2,\frac{3}{2}}$		
A++	0.333287-0.216064j	0.333287-0.216064j	7.61e-14
A+-	0.280891-0.182097j	0.280891-0.182097j	8.08e-14
A-+	-0.280891+0.182097j	-0.280891+0.182097j	8.08e-14
A--	0.333287-0.216064j	0.333287-0.216064j	7.61e-14
ArL(2000)1	$\mathcal{H}^{\text{LS,production}}_{L(2000),0,\frac{1}{2}}$		
A++	1.036314+1.105950j	1.036314+1.105950j	1.14e-15
A+-	0.024928+0.026603j	0.024928+0.026603j	7.76e-14
A-+	-0.024928-0.026603j	-0.024928-0.026603j	7.71e-14
A--	1.036314+1.105950j	1.036314+1.105950j	1.24e-15
ArL(2000)2	$\mathcal{H}^{\text{LS,production}}_{L(2000),1,\frac{1}{2}}$		
A++	-0.529297-0.564863j	-0.529297-0.564863j	1.87e-14
A+-	-1.009471-1.077303j	-1.009471-1.077303j	2.34e-14
A-+	-1.009471-1.077303j	-1.009471-1.077303j	2.34e-14
A--	0.529297+0.564863j	0.529297+0.564863j	1.87e-14

CHAPTER THREE

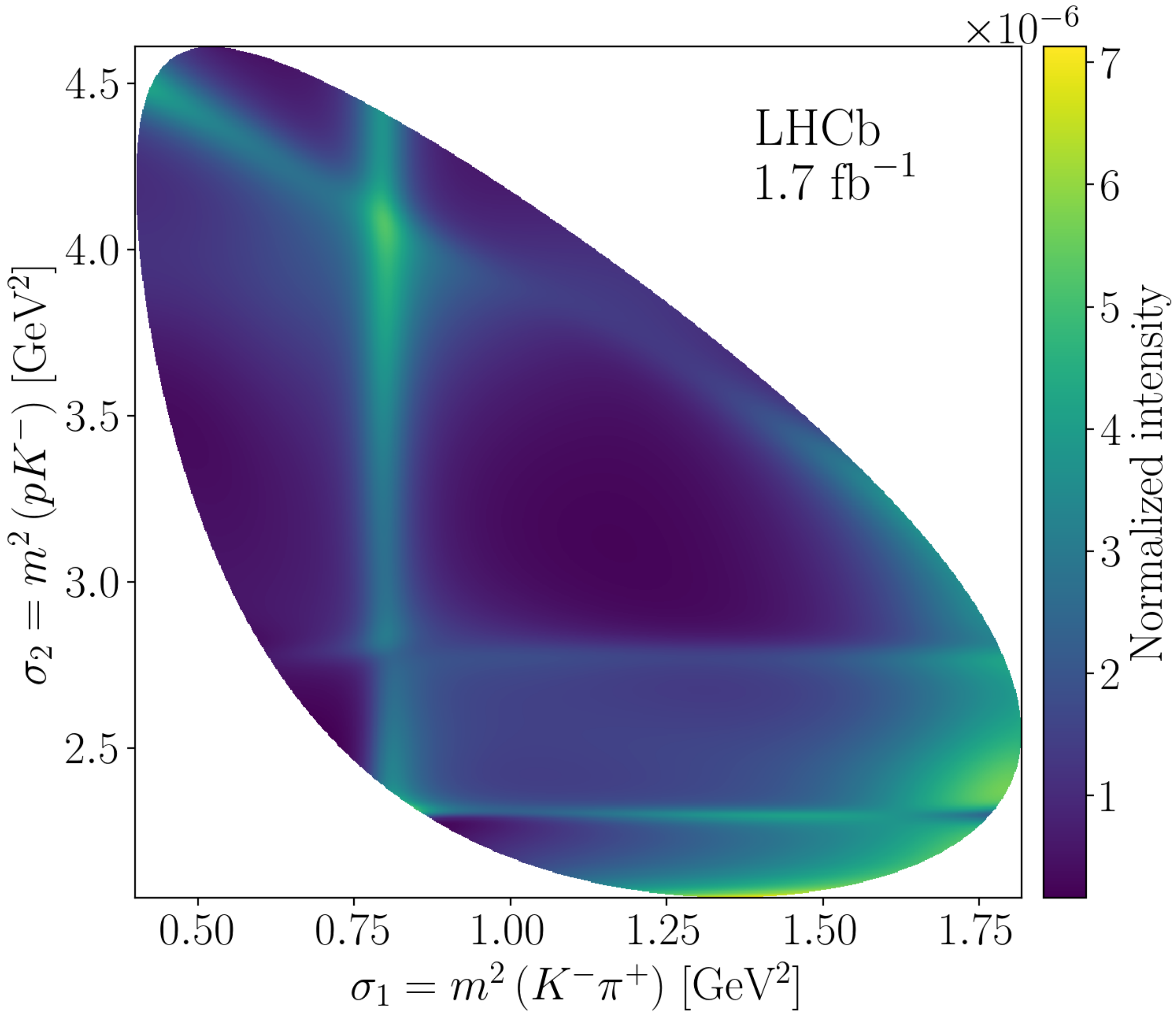
INTENSITY DISTRIBUTION

The complete intensity expression contains **43,198 mathematical operations**.

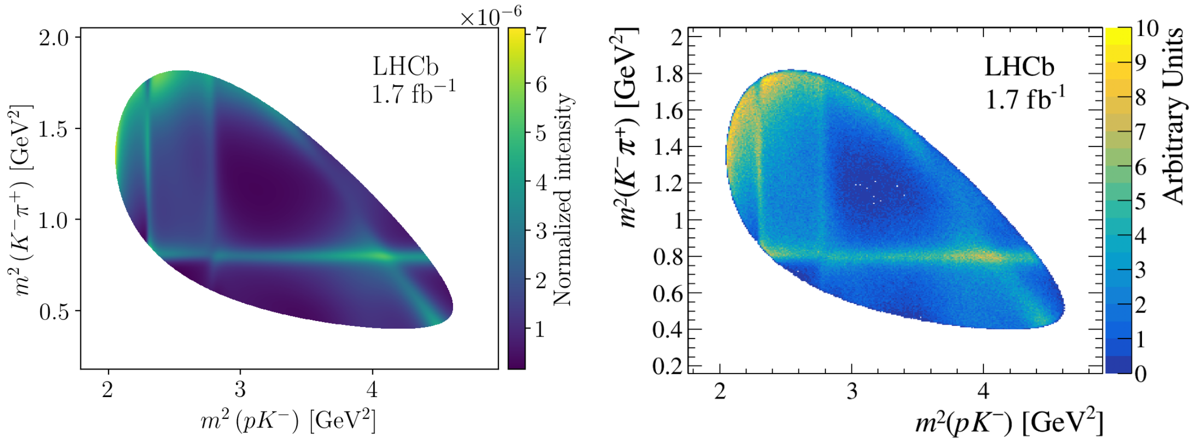
3.1 Definition of free parameters

After substituting the parameters that are not production couplings, the total intensity expression contains **9,516 operations**.

3.2 Distribution



Comparison with Figure 2 from the original LHCb study [1]:




```
<Figure size 1200x500 with 2 Axes>
```

3.3 Decay rates

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
<Figure size 900x900 with 1 Axes>
```

3.4 Dominant decays

```
<Figure size 910x700 with 1 Axes>
```

```
<Figure size 900x900 with 1 Axes>
```

CHAPTER

FOUR

POLARIMETER VECTOR FIELD

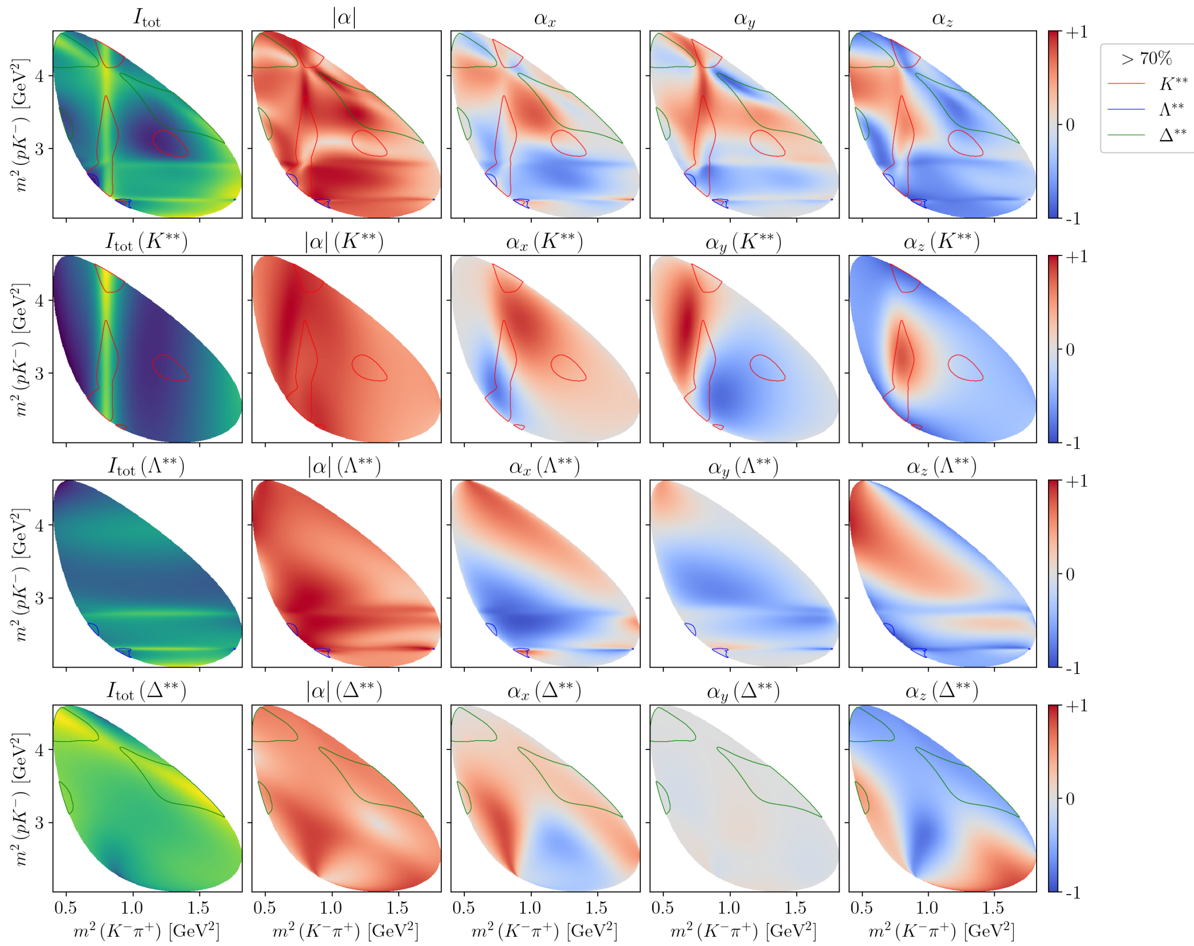
Final state IDs:

1. p
2. π^+
3. K^-

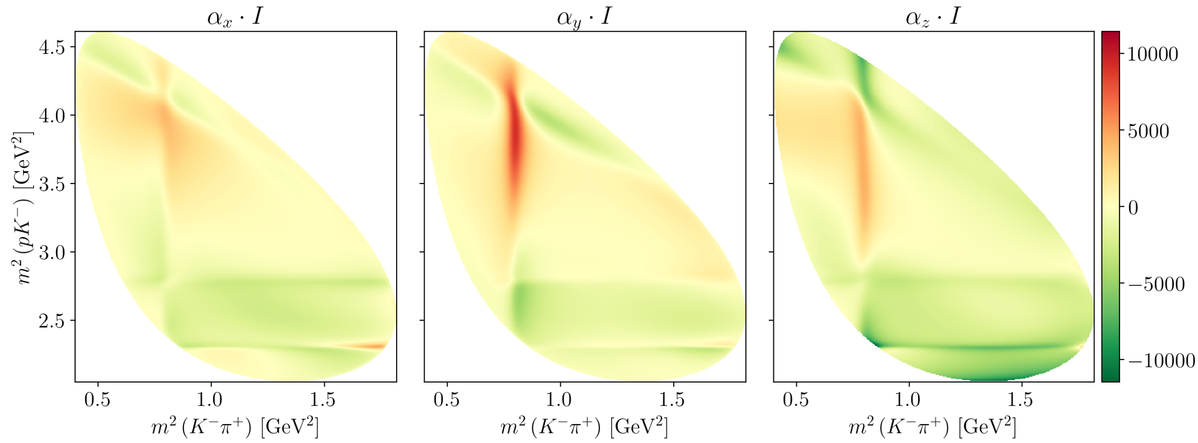
Sub-system definitions:

1. $K^{**} \to \pi^+ K^-$
2. $\Lambda^{**} \to p K^-$
3. $\Delta^{**} \to p \pi^+$

4.1 Dominant contributions




```
CPU times: user 44.7 s, sys: 2.04 s, total: 46.8 s
Wall time: 51.9 s
```



4.2 Total polarimetry vector field

```
  0%|          | 0/3 [00:00<?, ?it/s]
```

```
<IPython.core.display.SVG object>
```

```
<IPython.core.display.SVG object>
```

```
<IPython.core.display.SVG object>
```

4.3 Aligned vector fields per chain

```
<Figure size 1300x500 with 4 Axes>
```

```
<Figure size 1300x900 with 8 Axes>
```

```
<Figure size 1300x500 with 4 Axes>
```

```
<Figure size 1300x450 with 4 Axes>
```

CHAPTER

FIVE

UNCERTAINTIES

5.1 Model loading

Of the 18 models, there are 9 with a unique expression tree.

Show number of mathematical operations per model

		Model description	n ops.
0		Default amplitude model	$43,198$
1	$=0$	Alternative amplitude model with K(892) with free mass and width	$43,198$
2	$=0$	Alternative amplitude model with L(1670) with free mass and width	$43,198$
3	$=0$	Alternative amplitude model with L(1690) with free mass and width	$43,198$
4	$=0$	Alternative amplitude model with D(1232) with free mass and width	$43,198$
5	$=0$	Alternative amplitude model with L(1600), D(1600), D(1700) with free mass and width	$43,198$
6	$=0$	Alternative amplitude model with free L(1405) Flatt'e widths, indicated as G1 (pK channel) and G2 (Sigmapi)	$43,198$
7		Alternative amplitude model with L(1800) contribution added with free mass and width	$44,222$
8		Alternative amplitude model with L(1810) contribution added with free mass and width	$46,782$
9		Alternative amplitude model with D(1620) contribution added with free mass and width	$44,222$
10		Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(700) contribution	$43,470$
11	$=0$	Alternative amplitude model with K(700) with free mass and width	$43,198$
12		Alternative amplitude model with K(1410) contribution added with mass and width from PDG2020	$46,780$
13		Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(1430) contribution	$43,470$
14	$=0$	Alternative amplitude model with K(1430) with free width	$43,198$
15		Alternative amplitude model with an additional overall exponential form factor exp(-alpha q^2) multiplying Bugg lineshapes. The exponential parameter is indicated as ``alpha''	$43,582$
16	$=0$	Alternative amplitude model with free radial parameter d for the Lc resonance, indicated as dLc	$43,198$
17		Alternative amplitude model obtained using LS couplings	$110,839$

5.2 Statistical uncertainties

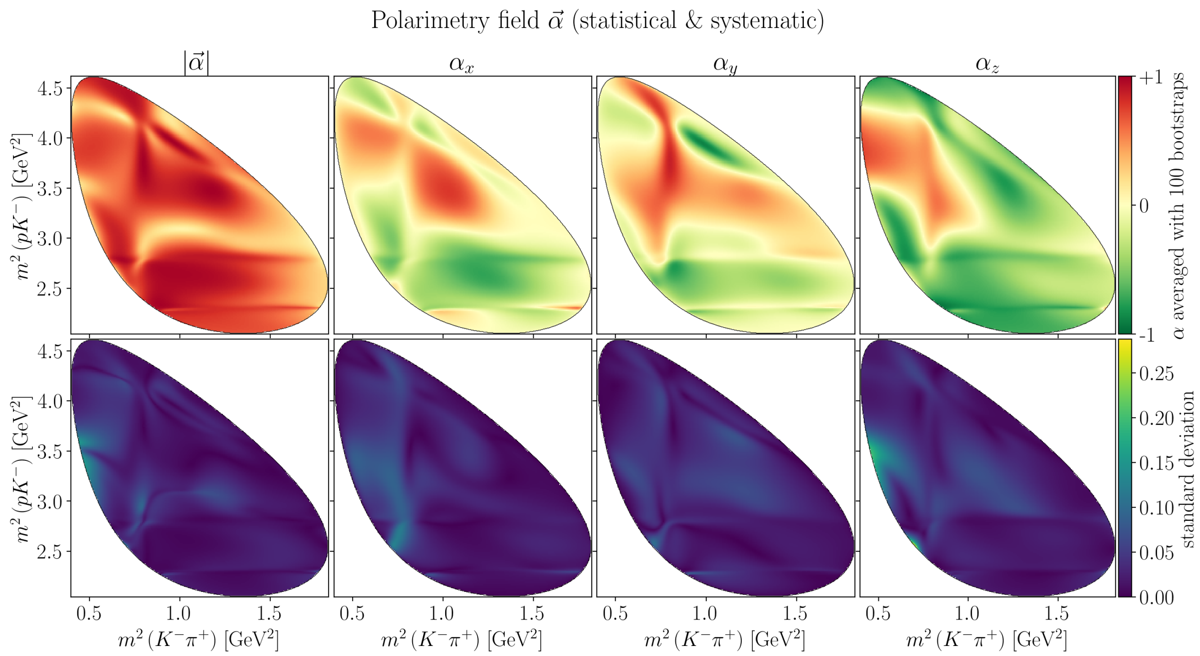
5.2.1 Parameter bootstrapping

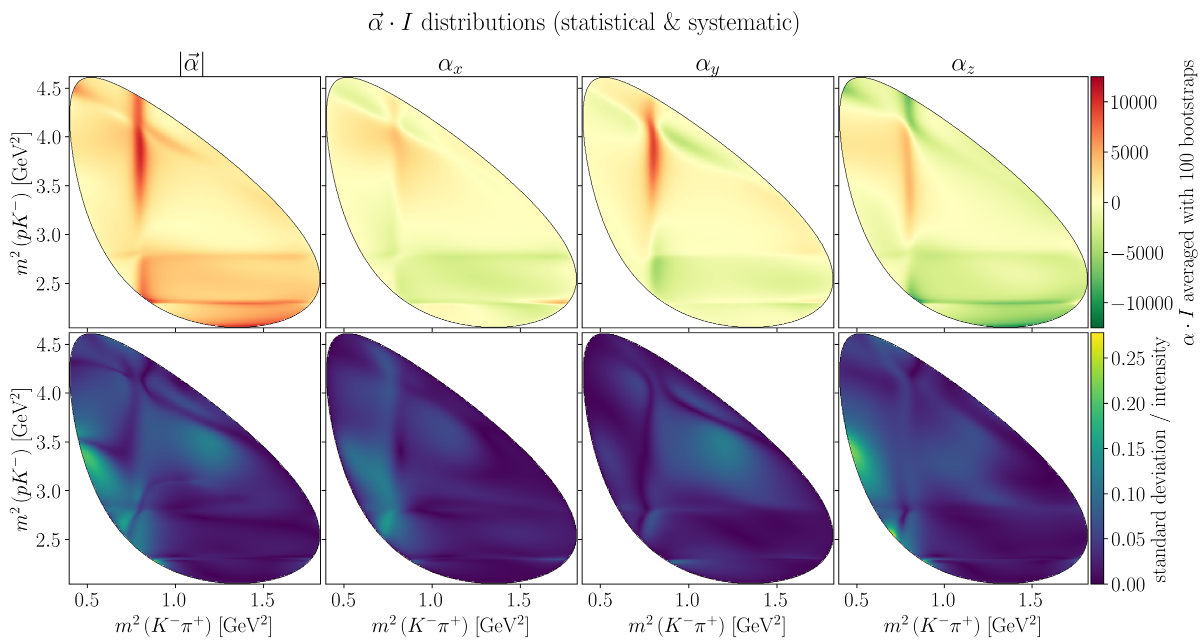
```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

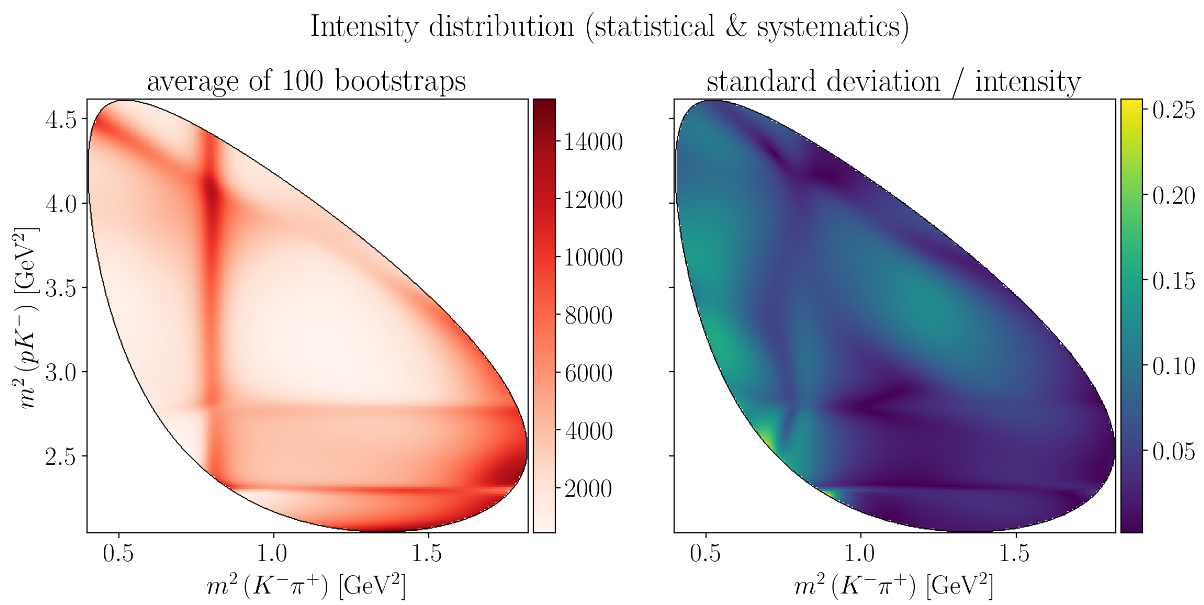
5.2.2 Mean and standard deviations

```
(100, 100000)
```

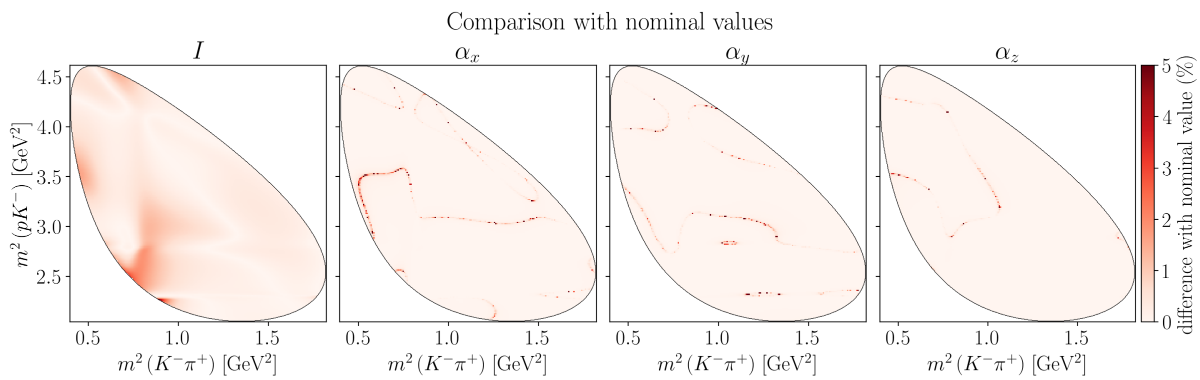
5.2.3 Distributions







5.2.4 Comparison with nominal values



5.3 Systematic uncertainties

5.3.1 Mean and standard deviations

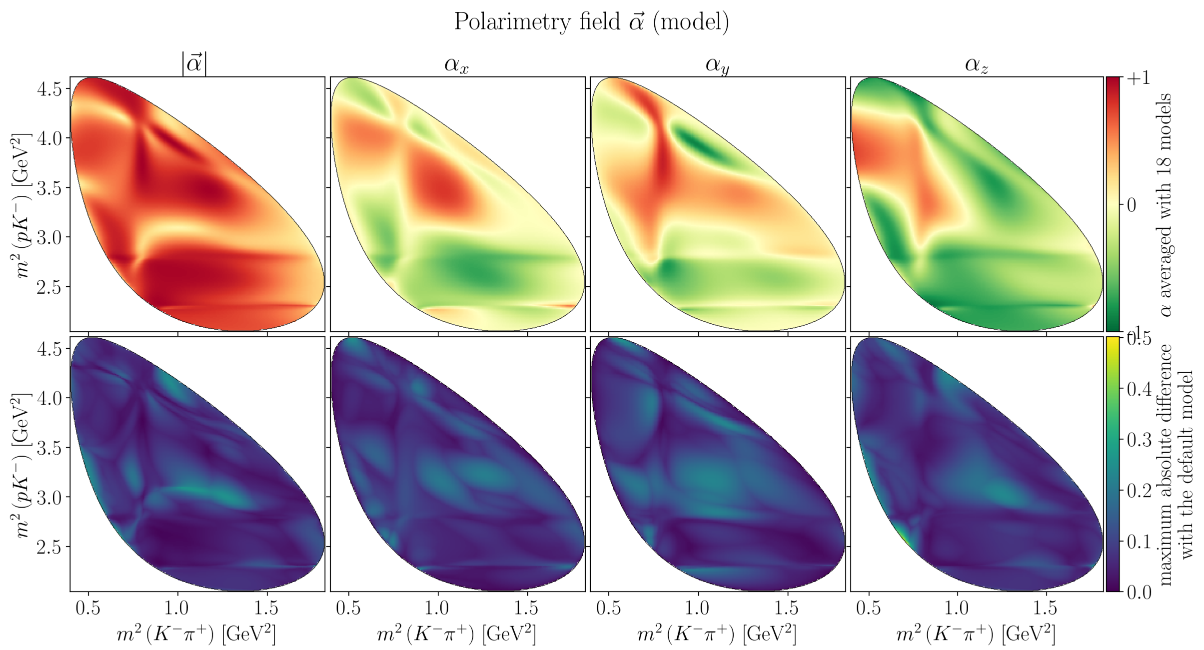
```
(18, 100000)
```

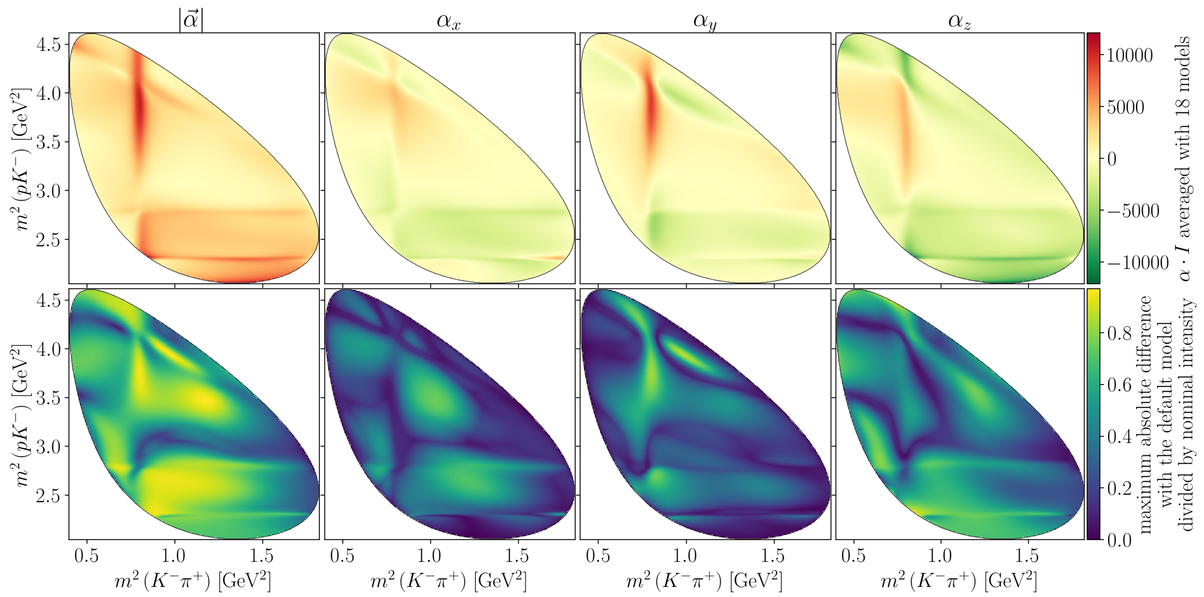
5.3.2 Distributions

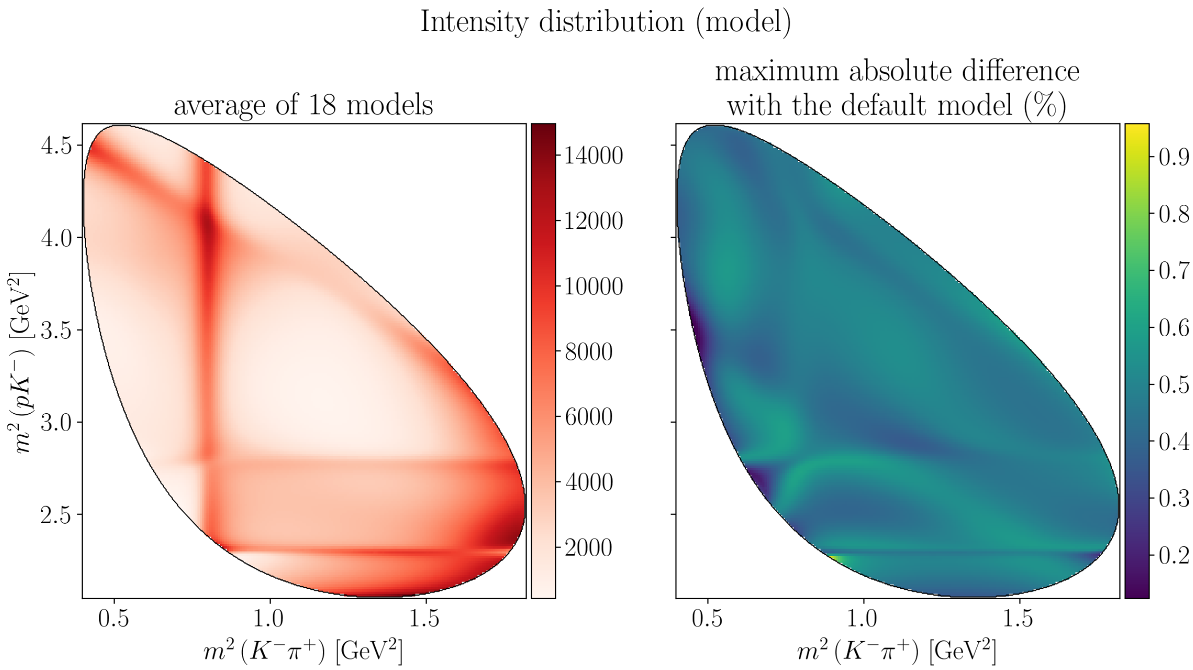
```
<Figure size 2000x1600 with 17 Axes>
```

```
<Figure size 2000x1600 with 17 Axes>
```

```
<Figure size 2000x1600 with 17 Axes>
```


Polarimetry field $\vec{\alpha}$ (model)
$|\vec{\alpha}|$
α_x
α_y
α_z
α averaged with 18 models
maximum absolute difference with the default model
$m^2(pK^-)$ [GeV^2]
$m^2(K^-\pi^+)$ [GeV^2]


$|\vec{\alpha}|$
α_x
α_y
α_z
$\alpha \cdot I$ averaged with 18 models
maximum absolute difference with the default model divided by nominal intensity
$m^2(pK^-)$ [GeV^2]
$m^2(K^-\pi^+)$ [GeV^2]



5.4 Uncertainty on polarimetry

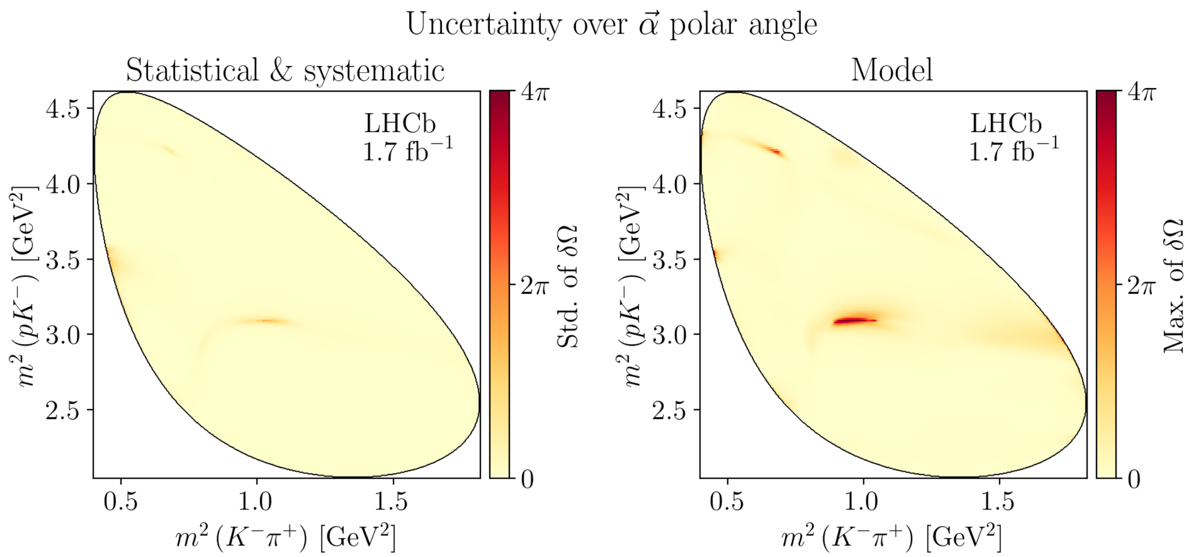
For each bootstrap or alternative model i, we compute the angle between each aligned polarimeter vector $\vec{\alpha}_i$ and the one from the nominal model, $\vec{\alpha}_0$:

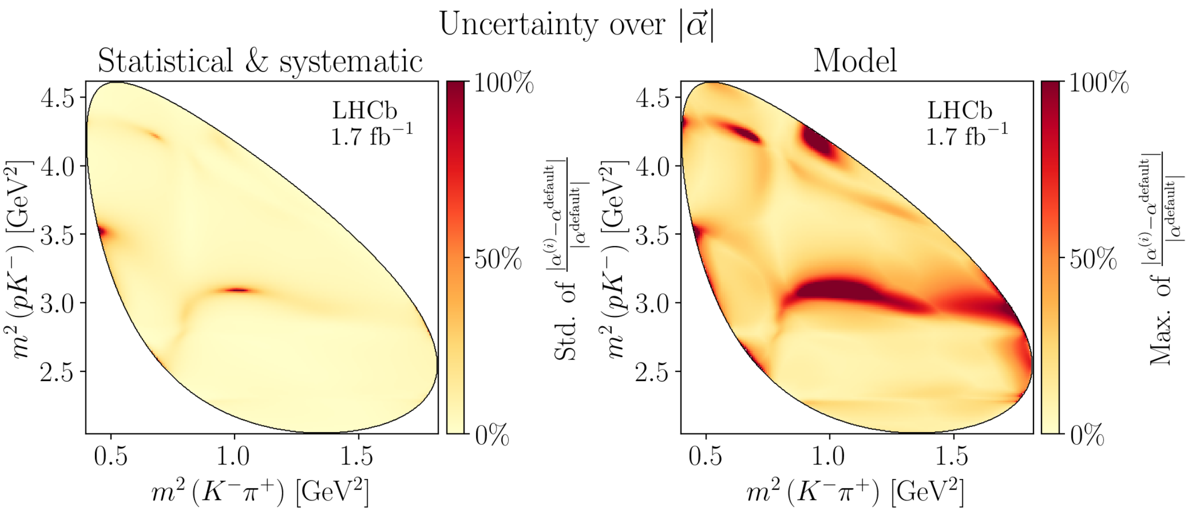
$$\cos\theta_i = \frac{\vec{\alpha}_i \cdot \vec{\alpha}_0}{|\alpha_i||\alpha_0|}.$$

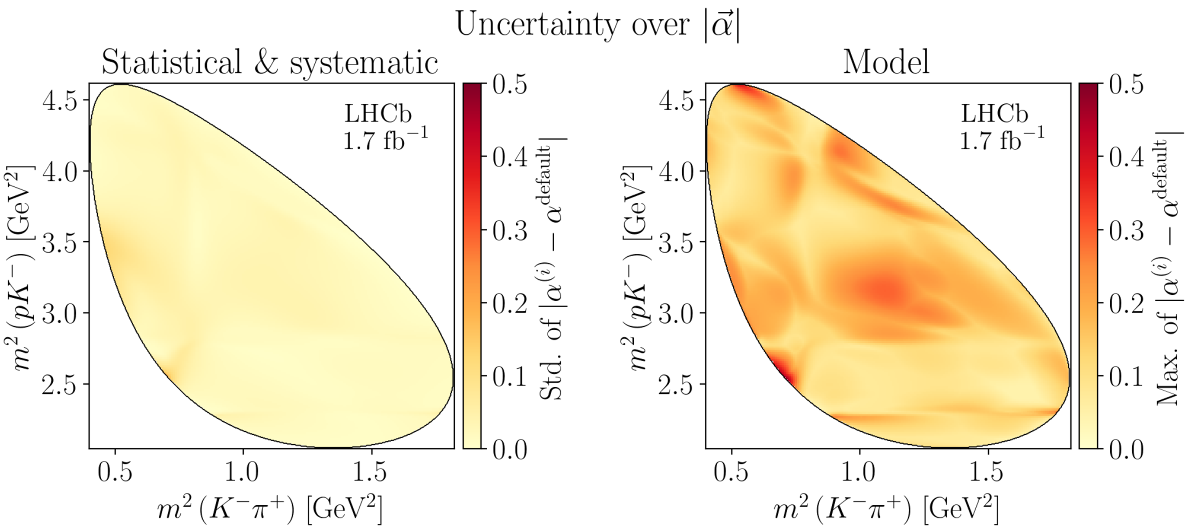
The solid angle can then be computed as:

$$\delta\Omega = \int_0^{2\pi}\int_0^{\theta} \mathrm{d}\phi\,\mathrm{d}\cos\theta = 2\pi\left(1-\cos\theta\right).$$

The statistical uncertainty is given by taking the standard deviation on the $\delta\Omega$ distribution and the systematic uncertainty is given by taking finding $\theta_\text{max} = \max\theta_i$ and computing $\delta\Omega_\text{max}$ from that.







5.5 Decay rates

Resonance	Decay rate	LHCb
$\Lambda(1405)$	$7.78 \pm 0.43^{+3.01}_{-2.53}$	$7.7 \pm 0.2 \pm 3.0$
$\Lambda(1520)$	$1.91 \pm 0.10^{+0.04}_{-0.24}$	$1.86 \pm 0.09 \pm 0.23$
$\Lambda(1600)$	$5.16 \pm 0.28^{+0.50}_{-1.93}$	$5.2 \pm 0.2 \pm 1.9$
$\Lambda(1670)$	$1.15 \pm 0.04^{+0.06}_{-0.29}$	$1.18 \pm 0.06 \pm 0.32$
$\Lambda(1690)$	$1.16 \pm 0.01^{+0.06}_{-0.33}$	$1.19 \pm 0.09 \pm 0.34$
$\Lambda(2000)$	$9.55 \pm 0.67^{+0.83}_{-2.26}$	$9.58 \pm 0.27 \pm 0.93$
$\Delta(1232)$	$28.73 \pm 1.34^{+1.76}_{-0.79}$	$28.6 \pm 0.29 \pm 0.76$
$\Delta(1600)$	$4.50 \pm 0.51^{+0.93}_{-1.40}$	$4.5 \pm 0.3 \pm 1.5$
$\Delta(1700)$	$3.89 \pm 0.07^{+0.94}_{-0.48}$	$3.9 \pm 0.2 \pm 0.94$
$K(700)$	$2.99 \pm 0.20^{+0.91}_{-0.59}$	$3.02 \pm 0.16 \pm 0.92$
$K(892)$	$21.95 \pm 1.24^{+0.59}_{-0.70}$	$22.14 \pm 0.23 \pm 0.64$
$K(1430)$	$14.70 \pm 0.80^{+2.78}_{-2.67}$	$14.7 \pm 0.6 \pm 2.7$

Resonance	1	2	3	4	5	6	7	8	9	10	11	12	13
$\Lambda(1405)$	+0.11	−0.14	−0.01	−0.33	−0.99	+3.01	−2.53	−0.66	−1.58	−0.43	−0.01	−1.97	−0.11
$\Lambda(1520)$	+0.03	+0.00	+0.01	+0.01	−0.24	−0.01	−0.04	−0.08	−0.06	−0.06	+0.04	−0.15	−0.00
$\Lambda(1600)$	−0.02	−0.09	+0.13	+0.22	+0.50	−0.09	−0.30	+0.23	−1.93	−0.46	+0.12	−1.85	−0.12
$\Lambda(1670)$	−0.01	+0.06	+0.03	+0.01	−0.01	−0.12	−0.29	−0.03	−0.11	+0.05	−0.01	−0.03	+0.01
$\Lambda(1690)$	+0.00	−0.00	+0.04	−0.13	+0.01	−0.06	−0.04	−0.26	−0.33	−0.08	−0.04	+0.06	+0.01
$\Lambda(2000)$	+0.05	+0.10	−0.08	−0.09	+0.08	−0.85	−2.26	+0.83	−0.93	+0.31	−0.23	−0.86	+0.35
$\Delta(1232)$	−0.27	+0.02	+0.31	+1.76	−0.44	−0.14	+0.49	−0.63	−0.77	+0.53	−0.31	+0.65	+0.10
$\Delta(1600)$	+0.33	−0.10	−0.15	−0.28	+0.59	−0.38	−1.40	−0.29	+0.93	+0.03	+0.05	−0.58	+0.07
$\Delta(1700)$	−0.01	+0.03	−0.13	+0.07	+0.39	−0.48	−0.15	+0.82	+0.94	+0.18	+0.05	+0.75	+0.03
$K(700)$	+0.17	−0.02	+0.04	+0.75	+0.62	−0.59	−0.31	−0.06	+0.91	+0.56	+0.25	+0.42	+0.10
$K(892)$	−0.53	−0.02	−0.12	−0.46	−0.58	+0.55	+0.59	−0.06	+0.18	+0.28	−0.16	+0.25	−0.01
$K(1430)$	−0.29	+0.07	−0.50	−0.03	+0.18	−0.76	+2.78	+2.40	−2.67	+1.29	+0.23	−2.27	+0.91

- **0**: Default amplitude model
- **1**: Alternative amplitude model with K(892) with free mass and width
- **2**: Alternative amplitude model with L(1670) with free mass and width
- **3**: Alternative amplitude model with L(1690) with free mass and width
- **4**: Alternative amplitude model with D(1232) with free mass and width
- **5**: Alternative amplitude model with L(1600), D(1600), D(1700) with free mass and width
- **6**: Alternative amplitude model with free L(1405) Flatt'e widths, indicated as G1 (pK channel) and G2 (Sigmapi)
- **7**: Alternative amplitude model with L(1800) contribution added with free mass and width
- **8**: Alternative amplitude model with L(1810) contribution added with free mass and width
- **9**: Alternative amplitude model with D(1620) contribution added with free mass and width
- **10**: Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(700) contribution
- **11**: Alternative amplitude model with K(700) with free mass and width
- **12**: Alternative amplitude model with K(1410) contribution added with mass and width from PDG2020
- **13**: Alternative amplitude model in which a Relativistic Breit-Wigner is used for the K(1430) contribution
- **14**: Alternative amplitude model with K(1430) with free width
- **15**: Alternative amplitude model with an additional overall exponential form factor exp(-alpha q^2) multiplying Bugg lineshapes. The exponential parameter is indicated as ``alpha"
- **16**: Alternative amplitude model with free radial parameter d for the Lc resonance, indicated as dLc
- **17**: Alternative amplitude model obtained using LS couplings

5.6 Average polarimetry values

The components of the **averaged polarimeter vector** $\overline{\alpha}$ are defined as:

$$\overline{\alpha}_j = \int I_0(\tau)\,\alpha_j(\tau)\,\mathrm{d}^n\tau \,\Big/ \int I_0(\tau)\;\mathrm{d}^n\tau$$

The averages of the norm of $\vec{\alpha}$ are computed as follows:

- $\left|\overline{\alpha}\right| = \sqrt{\overline{\alpha_x}^2 + \overline{\alpha_y}^2 + \overline{\alpha_z}^2}$, with the statistical uncertainties added in quadrature and the systematic uncertainties by taking the same formula on the extrema values of each $\overline{\alpha_j}$
- $\overline{|\alpha|} = \sqrt{\int I_0(\tau)\left|\vec{\alpha}(\tau)\right|^2 \mathrm{d}^n\tau \,/ \int I_0(\tau)\;\mathrm{d}^n\tau}$

Cartesian coordinates:

$$
\begin{array}{rcl}
\overline{\alpha_x} & = & \left(-62.6 \pm 4.5^{+8.4}_{-14.8}\right) \times 10^{-3} \\
\overline{\alpha_y} & = & \left(+8.9 \pm 8.9^{+9.1}_{-12.7}\right) \times 10^{-3} \\
\overline{\alpha_z} & = & \left(-278.0 \pm 23.7^{+12.6}_{-40.4}\right) \times 10^{-3} \\
\overline{|\alpha|} & = & \left(669.4 \pm 9.3^{+15.3}_{-10.4}\right) \times 10^{-3}
\end{array}
$$

Polar coordinates:

$$
\begin{array}{rcl}
\theta\left(\vec{\alpha}\right) & = & \arccos\left(\alpha_z \big/ |\alpha|\right) \\
\phi\left(\vec{\alpha}\right) & = & \pi - \mathrm{atan2}\left(\alpha_y, -\alpha_x\right)
\end{array}
$$

$$
\begin{array}{rcl}
|\overline{\alpha}| & = & \left(+285.1 \pm 24.0^{+37.9}_{-13.8}\right) \times 10^{-3} \\
\theta\left(\overline{\alpha}\right) & = & +2.92 \pm 0.01^{+0.05}_{-0.04}\ \mathrm{rad} \\
 & = & \left(+0.929 \pm 0.002^{+0.017}_{-0.011}\right) \times \pi \\
\phi\left(\overline{\alpha}\right) & = & +3.00 \pm 0.14^{+0.21}_{-0.09}\ \mathrm{rad} \\
 & = & \left(+0.955 \pm 0.045^{+0.067}_{-0.028}\right) \times \pi
\end{array}
$$

Averaged polarimeter values for each model (and the difference with the nominal model):

Model	$\overline{\alpha}_x$	$\overline{\alpha}_y$	$\overline{\alpha}_z$	$\overline{\lvert\alpha\rvert}$	$\Delta\overline{\alpha}_x$	$\Delta\overline{\alpha}_y$	$\Delta\overline{\alpha}_z$	$\Delta\overline{\lvert\alpha\rvert}$
0	-62.6	$+8.9$	-278.0	669.4				
1	-61.6	$+8.5$	-279.4	670.7	$+1.0$	-0.4	-1.4	$+1.3$
2	-62.9	$+9.1$	-278.4	669.8	-0.3	$+0.2$	-0.5	$+0.4$
3	-58.4	$+7.4$	-276.2	667.7	$+4.2$	-1.5	$+1.8$	-1.6
4	-69.3	$+9.5$	-277.2	666.9	-6.6	$+0.6$	$+0.8$	-2.5
5	-70.7	$+9.6$	-277.4	668.7	-8.0	$+0.8$	$+0.6$	-0.6
6	-69.7	$+9.1$	-281.7	673.0	-7.1	$+0.2$	-3.8	$+3.7$
7	-77.4	$+18.0$	-305.4	671.4	-14.8	$+9.1$	-27.5	$+2.1$
8	-55.8	$+10.9$	-284.6	675.5	$+6.8$	$+2.0$	-6.7	$+6.1$
9	-66.9	$+4.4$	-290.4	672.8	-4.3	-4.5	-12.4	$+3.5$
10	-56.4	$+2.4$	-265.4	659.0	$+6.2$	-6.5	$+12.6$	-10.4
11	-64.7	$+9.3$	-278.6	670.4	-2.1	$+0.4$	-0.6	$+1.0$
12	-75.1	$+1.8$	-283.4	663.5	-12.5	-7.1	-5.4	-5.8
13	-61.8	$+8.1$	-277.3	668.8	$+0.9$	-0.8	$+0.7$	-0.6
14	-62.2	$+8.7$	-277.6	669.2	$+0.5$	-0.2	$+0.4$	-0.2
15	-54.2	-3.8	-318.4	684.6	$+8.4$	-12.7	-40.4	$+15.3$
16	-62.1	$+8.2$	-278.1	669.5	$+0.5$	-0.7	-0.1	$+0.2$
17	-58.1	$+12.1$	-278.6	666.5	$+4.5$	$+3.2$	-0.6	-2.9

Model	$10^3 \cdot \lvert\overline{\alpha}\rvert$	$\theta\left(\overline{\alpha}\right)/\pi$	$\phi\left(\overline{\alpha}\right)/\pi$	$10^3 \cdot \Delta\lvert\overline{\alpha}\rvert$	$\Delta\theta\left(\overline{\alpha}\right)/\pi$	$\Delta\phi\left(\overline{\alpha}\right)/\pi$
0	$+285.1$	$+0.929$	$+0.955$			
1	$+286.2$	$+0.930$	$+0.956$	$+1.1$	$+0.001$	$+0.001$
2	$+285.6$	$+0.929$	$+0.954$	$+0.5$	-0.000	-0.001
3	$+282.4$	$+0.933$	$+0.960$	-2.7	$+0.004$	$+0.005$
4	$+285.8$	$+0.921$	$+0.956$	$+0.8$	-0.007	$+0.001$
5	$+286.4$	$+0.920$	$+0.957$	$+1.4$	-0.009	$+0.002$
6	$+290.4$	$+0.922$	$+0.959$	$+5.3$	-0.007	$+0.004$
7	$+315.6$	$+0.919$	$+0.927$	$+30.5$	-0.010	-0.028
8	$+290.3$	$+0.937$	$+0.939$	$+5.2$	$+0.008$	-0.017
9	$+298.0$	$+0.928$	$+0.979$	$+12.9$	-0.001	$+0.024$
10	$+271.3$	$+0.933$	$+0.987$	-13.8	$+0.004$	$+0.031$
11	$+286.2$	$+0.927$	$+0.955$	$+1.1$	-0.002	-0.000
12	$+293.2$	$+0.918$	$+0.992$	$+8.1$	-0.011	$+0.037$
13	$+284.2$	$+0.930$	$+0.958$	-0.9	$+0.001$	$+0.003$
14	$+284.6$	$+0.929$	$+0.956$	-0.5	$+0.000$	$+0.001$
15	$+323.0$	$+0.946$	$+1.022$	$+37.9$	$+0.017$	$+0.067$
16	$+285.1$	$+0.929$	$+0.958$	-0.0	$+0.001$	$+0.003$
17	$+284.8$	$+0.933$	$+0.935$	-0.2	$+0.004$	-0.021

Tip: These values can be downloaded in serialized JSON format under *Exported distributions* (page 35).

```
<Figure size 1100x500 with 2 Axes>
```

```
<Figure size 1100x500 with 2 Axes>
```

```
<Figure size 900x500 with 2 Axes>
```

```
<Figure size 900x500 with 2 Axes>
```

Tip: A potential explanation for the xz-correlation may be found in Section *XZ-correlations* (page 37).

5.7 Exported distributions

The polarimetry fields are computed for each parameter bootstrap (statistics & systematics) and for each model on lc2pkpi-polarimetry.docs.cern.ch/uncertainties.html. All combined fields can be downloaded as single compressed TAR file under lc2pkpi-polarimetry.docs.cern.ch/_static/export/polarimetry-field.json and as a single JSON file under lc2pkpi-polarimetry.docs.cern.ch/_static/export/polarimetry-field.tar.gz.

Tip: See *Import and interpolate* (page 47) for how to use these grids in an an analysis and see *Determination of polarization* (page 52) for how to use these fields to determine the polarization from a measured distribution.

CHAPTER

SIX

AVERAGE POLARIMETER PER RESONANCE

6.1 Computations

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

6.2 Result and comparison

LHCb values are taken from the original study [1]:

	this study	LHCb	1	2	3	4	5	6	7	8	9	10
$K(700)$	$+63 \pm 78^{+238}_{-235}$	$+60 \pm 660 \pm 240$	-5	-14	-55	-113	-100	$+57$	-176	-235	$+238$	$+96$
$K(892)$	$+29 \pm 15^{+31}_{-17}$		$+2$	-0	$+2$	-9	-17	$+2$	-5	$+23$	$+31$	-8
$K(1430)$	$-339 \pm 28^{+139}_{-102}$	$-340 \pm 30 \pm 140$	$+3$	$+3$	-1	-2	$+45$	$+102$	$+125$	-9	-102	$+139$
$\Lambda(1405)$	$+580 \pm 31^{+278}_{-122}$	$-580 \pm 50 \pm 280$	$+14$	-7	$+3$	$+31$	-3	-30	-122	-22	$+124$	-64
$\Lambda(1520)$	$+925 \pm 8^{+16}_{-84}$	$-925 \pm 25 \pm 84$	$+7$	$+2$	$+2$	$+16$	-34	$+2$	$+8$	$+11$	$+7$	-3
$\Lambda(1600)$	$+199 \pm 51^{+499}_{-428}$	$-200 \pm 60 \pm 500$	$+10$	-5	$+14$	-5	$+21$	$+138$	$+100$	$+499$	-428	-140
$\Lambda(1670)$	$+817 \pm 15^{+73}_{-46}$	$-817 \pm 42 \pm 73$	$+9$	-10	$+12$	$+70$	-41	-5	$+73$	$+30$	$+47$	-46
$\Lambda(1690)$	$+958 \pm 8^{+27}_{-35}$	$-958 \pm 20 \pm 27$	-3	$+6$	-12	-35	-14	$+22$	$+27$	-20	$+3$	-4
$\Lambda(2000)$	$-573 \pm 9^{+124}_{-191}$	$+570 \pm 30 \pm 190$	$+9$	-1	$+12$	$+47$	-24	-45	-191	$+58$	$+85$	$+78$
$\Delta(1232)$	$+548 \pm 8^{+36}_{-27}$	$-548 \pm 14 \pm 36$	$+9$	$+0$	-9	-14	$+17$	-1	$+10$	$+36$	$+5$	-11
$\Delta(1600)$	$-502 \pm 9^{+162}_{-112}$	$+500 \pm 50 \pm 170$	$+19$	$+10$	$+6$	$+107$	-112	$+115$	$+88$	$+49$	$+162$	$+5$
$\Delta(1700)$	$+216 \pm 18^{+42}_{-75}$	$-216 \pm 36 \pm 75$	$+40$	$+4$	-0	-19	-2	$+23$	$+16$	$+42$	$+23$	-75

6.3 Distribution analysis

6.3.1 XZ-correlations

It follows from the definition of $\vec{\alpha}$ for a single resonance that:

$$\begin{aligned}\alpha_x &= |\vec{\alpha}| \int I_0 \sin\left(\zeta^0\right)\, \mathrm{d}\tau \Big/ \int I_0\, \mathrm{d}\tau \\ \alpha_z &= |\vec{\alpha}| \int I_0 \cos\left(\zeta^0\right)\, \mathrm{d}\tau \Big/ \int I_0\, \mathrm{d}\tau\end{aligned}$$

This means that the correlation if 100% if I_0 does not change in the bootstrap. This may explain the xz-correlation observed for $\overline{\alpha}$ over the complete decay as reported in *Average polarimetry values* (page 33).

$$I_{L(2000)} = \frac{155.425\sigma_2^2}{\left|\sigma_2(\sigma_2-3.953)+0.79i\sqrt{0.445\sigma_2^2-\sigma_2+0.18}\right|^2}$$

$$
\begin{aligned}
\alpha_{x,L(2000)} &= -0.572 \sin\left(\zeta^0_{2(1)}\right) \\
\alpha_{z,L(2000)} &= -0.572 \cos\left(\zeta^0_{2(1)}\right)
\end{aligned}
$$

```
<Figure size 1200x800 with 12 Axes>
```

```
<Figure size 1200x800 with 12 Axes>
```

Tip: The following plots are interactive and can best be viewed on lc2pkpi-polarimetry.docs.cern.ch.

CHAPTER

SEVEN

APPENDIX

7.1 Dynamics lineshapes

$$
\begin{aligned}
F_{L}\left(z\right) &= \begin{cases} 1 & \text{for}\: L = 0 \\ \frac{1}{\sqrt{z^{2} + 1}} & \text{for}\: L = 1 \\ \frac{1}{\sqrt{z^{4} + 3 z^{2} + 9}} & \text{for}\: L = 2 \end{cases} \\
\lambda\left(x, y, z\right) &= x^{2} - 2 x y - 2 x z + y^{2} - 2 y z + z^{2}
\end{aligned}
$$

$$
\begin{aligned}
p_{m_{i},m_{j}}\left(s\right) &= \frac{\sqrt{\lambda\left(s, m_{i}^{2}, m_{j}^{2}\right)}}{2 \sqrt{s}} \\
q_{m_{0},m_{k}}\left(s\right) &= \frac{\sqrt{\lambda\left(s, m_{0}^{2}, m_{k}^{2}\right)}}{2 m_{0}}
\end{aligned}
$$

$$
\Gamma\left(s\right) = \Gamma_{0} \frac{m}{\sqrt{s}} \frac{F_{l_{R}}\left(R p_{m_{1},m_{2}}\left(s\right)\right)^{2}}{F_{l_{R}}\left(R p_{m_{1},m_{2}}\left(m^{2}\right)\right)^{2}} \left(\frac{p_{m_{1},m_{2}}\left(s\right)}{p_{m_{1},m_{2}}\left(m^{2}\right)}\right)^{2 l_{R} + 1}
$$

7.1.1 Relativistic Breit-Wigner

$$
\mathcal{R}\left(s\right) = \frac{\frac{F_{l_{R}}\left(R_\text{res} p_{m_{1},m_{2}}\left(s\right)\right)}{F_{l_{R}}\left(R_\text{res} p_{m_{1},m_{2}}\left(m^{2}\right)\right)} \frac{F_{l_{\Lambda_c}}\left(R_{\Lambda_c} q_{m_\text{top},m_\text{spectator}}\left(s\right)\right)}{F_{l_{\Lambda_c}}\left(R_{\Lambda_c} q_{m_\text{top},m_\text{spectator}}\left(m^{2}\right)\right)} \left(\frac{p_{m_{1},m_{2}}\left(s\right)}{p_{m_{1},m_{2}}\left(m^{2}\right)}\right)^{l_{R}} \left(\frac{q_{m_\text{top},m_\text{spectator}}\left(s\right)}{q_{m_\text{top},m_\text{spectator}}\left(m^{2}\right)}\right)^{l_{\Lambda_c}}}{m^{2} - i m \Gamma\left(s\right) - s}
$$

7.1.2 Bugg Breit-Wigner

$$
\begin{aligned}
\mathcal{R}_\text{Bugg}\left(m_{K\pi}^{2}\right) &= \frac{1}{- \frac{i \Gamma_{0} m_{0} \left(m_{K\pi}^{2} - s_{A}\right) e^{- \gamma m_{K\pi}^{2}}}{m_{0}^{2} - s_{A}} + m_{0}^{2} - m_{K\pi}^{2}} \\
s_{A} &= m_{K}^{2} - \frac{m_{\pi}^{2}}{2} \\
p_{m_{K},m_{\pi}}\left(m_{K\pi}^{2}\right) &= \frac{\sqrt{\lambda\left(m_{K\pi}^{2}, m_{K}^{2}, m_{\pi}^{2}\right)}}{2 \sqrt{m_{K\pi}^{2}}}
\end{aligned}
$$

One of the models uses a Bugg Breit-Wigner with an exponential factor:

$$
e^{- \alpha q_{m_{0},m_{1}}\left(s\right)^{2}} \mathcal{R}_\text{Bugg}\left(m_{K\pi}^{2}\right)
$$

7.1.3 Flatté for S-waves

$$\mathcal{R}_{\text{Flatté}}\left(s\right) = \frac{1}{m^{2}-im\left(\frac{\Gamma_{1}mp_{m_{11},m_{21}}\left(s\right)}{\sqrt{s}p_{m_{12},m_{22}}\left(m^{2}\right)}+\frac{\Gamma_{2}mp_{m_{12},m_{22}}\left(s\right)}{\sqrt{s}p_{m_{12},m_{22}}\left(m^{2}\right)}\right)-s}$$

7.2 DPD angles

Equation (A1) from [2]:

$$\begin{aligned}
\theta_{12} &= \operatorname{acos}\left(\frac{2\sigma_{3}(-m_{1}^{2}-m_{3}^{2}+\sigma_{2})-(m_{0}^{2}-m_{3}^{2}-\sigma_{3})(m_{1}^{2}-m_{2}^{2}+\sigma_{3})}{\sqrt{\lambda(m_{0}^{2},m_{3}^{2},\sigma_{3})}\sqrt{\lambda(\sigma_{3},m_{1}^{2},m_{2}^{2})}}\right)\\
\theta_{23} &= \operatorname{acos}\left(\frac{2\sigma_{1}(-m_{1}^{2}-m_{2}^{2}+\sigma_{3})-(m_{0}^{2}-m_{1}^{2}-\sigma_{1})(m_{2}^{2}-m_{3}^{2}+\sigma_{1})}{\sqrt{\lambda(m_{0}^{2},m_{1}^{2},\sigma_{1})}\sqrt{\lambda(\sigma_{1},m_{2}^{2},m_{3}^{2})}}\right)\\
\theta_{31} &= \operatorname{acos}\left(\frac{2\sigma_{2}(-m_{2}^{2}-m_{3}^{2}+\sigma_{1})-(m_{0}^{2}-m_{2}^{2}-\sigma_{2})(-m_{1}^{2}+m_{3}^{2}+\sigma_{2})}{\sqrt{\lambda(m_{0}^{2},m_{2}^{2},\sigma_{2})}\sqrt{\lambda(\sigma_{2},m_{3}^{2},m_{1}^{2})}}\right)
\end{aligned}$$

Equation (A3):

$$\begin{aligned}
\hat{\theta}_{3(1)} &= \operatorname{acos}\left(\frac{-2m_{0}^{2}(-m_{1}^{2}-m_{3}^{2}+\sigma_{2})+(m_{0}^{2}+m_{1}^{2}-\sigma_{1})(m_{0}^{2}+m_{3}^{2}-\sigma_{3})}{\sqrt{\lambda(m_{0}^{2},m_{1}^{2},\sigma_{1})}\sqrt{\lambda(m_{0}^{2},\sigma_{3},m_{3}^{2})}}\right)\\
\hat{\theta}_{1(2)} &= \operatorname{acos}\left(\frac{-2m_{0}^{2}(-m_{1}^{2}-m_{2}^{2}+\sigma_{3})+(m_{0}^{2}+m_{1}^{2}-\sigma_{1})(m_{0}^{2}+m_{2}^{2}-\sigma_{2})}{\sqrt{\lambda(m_{0}^{2},m_{2}^{2},\sigma_{2})}\sqrt{\lambda(m_{0}^{2},\sigma_{1},m_{1}^{2})}}\right)\\
\hat{\theta}_{2(3)} &= \operatorname{acos}\left(\frac{-2m_{0}^{2}(-m_{2}^{2}-m_{3}^{2}+\sigma_{1})+(m_{0}^{2}+m_{2}^{2}-\sigma_{2})(m_{0}^{2}+m_{3}^{2}-\sigma_{3})}{\sqrt{\lambda(m_{0}^{2},m_{3}^{2},\sigma_{3})}\sqrt{\lambda(m_{0}^{2},\sigma_{2},m_{2}^{2})}}\right)
\end{aligned}$$

Equations (A7):

$$\begin{aligned}
\zeta^{1}_{1(3)} &= \operatorname{acos}\left(\frac{2m_{1}^{2}(-m_{0}^{2}-m_{2}^{2}+\sigma_{2})+(m_{0}^{2}+m_{1}^{2}-\sigma_{1})(-m_{1}^{2}-m_{2}^{2}+\sigma_{3})}{\sqrt{\lambda(m_{0}^{2},m_{1}^{2},\sigma_{1})}\sqrt{\lambda(\sigma_{3},m_{1}^{2},m_{2}^{2})}}\right)\\
\zeta^{1}_{2(1)} &= \operatorname{acos}\left(\frac{2m_{1}^{2}(-m_{0}^{2}-m_{3}^{2}+\sigma_{3})+(m_{0}^{2}+m_{1}^{2}-\sigma_{1})(-m_{1}^{2}-m_{3}^{2}+\sigma_{2})}{\sqrt{\lambda(m_{0}^{2},m_{1}^{2},\sigma_{1})}\sqrt{\lambda(\sigma_{2},m_{1}^{2},m_{3}^{2})}}\right)\\
\zeta^{2}_{2(1)} &= \operatorname{acos}\left(\frac{2m_{2}^{2}(-m_{0}^{2}-m_{3}^{2}+\sigma_{3})+(m_{0}^{2}+m_{2}^{2}-\sigma_{2})(-m_{2}^{2}-m_{3}^{2}+\sigma_{1})}{\sqrt{\lambda(m_{0}^{2},m_{2}^{2},\sigma_{2})}\sqrt{\lambda(\sigma_{1},m_{2}^{2},m_{3}^{2})}}\right)\\
\zeta^{2}_{3(2)} &= \operatorname{acos}\left(\frac{2m_{2}^{2}(-m_{0}^{2}-m_{1}^{2}+\sigma_{1})+(m_{0}^{2}+m_{2}^{2}-\sigma_{2})(-m_{1}^{2}-m_{2}^{2}+\sigma_{3})}{\sqrt{\lambda(m_{0}^{2},m_{2}^{2},\sigma_{2})}\sqrt{\lambda(\sigma_{3},m_{2}^{2},m_{1}^{2})}}\right)\\
\zeta^{3}_{3(2)} &= \operatorname{acos}\left(\frac{2m_{3}^{2}(-m_{0}^{2}-m_{1}^{2}+\sigma_{1})+(m_{0}^{2}+m_{3}^{2}-\sigma_{3})(-m_{1}^{2}-m_{3}^{2}+\sigma_{2})}{\sqrt{\lambda(m_{0}^{2},m_{3}^{2},\sigma_{3})}\sqrt{\lambda(\sigma_{2},m_{3}^{2},m_{1}^{2})}}\right)\\
\zeta^{3}_{1(3)} &= \operatorname{acos}\left(\frac{2m_{3}^{2}(-m_{0}^{2}-m_{2}^{2}+\sigma_{2})+(m_{0}^{2}+m_{3}^{2}-\sigma_{3})(-m_{2}^{2}-m_{3}^{2}+\sigma_{1})}{\sqrt{\lambda(m_{0}^{2},m_{3}^{2},\sigma_{3})}\sqrt{\lambda(\sigma_{1},m_{3}^{2},m_{2}^{2})}}\right)
\end{aligned}$$

Equations (A10):

$$
\begin{aligned}
\zeta^1_{2(3)} &= \operatorname{acos}\left(\frac{2m_1^2(m_2^2+m_3^2-\sigma_1)+(-m_1^2-m_2^2+\sigma_3)(-m_1^2-m_3^2+\sigma_2)}{\sqrt{\lambda(\sigma_2,m_3^2,m_1^2)}\sqrt{\lambda(\sigma_3,m_1^2,m_2^2)}}\right) \\
\zeta^2_{3(1)} &= \operatorname{acos}\left(\frac{2m_2^2(m_1^2+m_3^2-\sigma_2)+(-m_1^2-m_2^2+\sigma_3)(-m_2^2-m_3^2+\sigma_1)}{\sqrt{\lambda(\sigma_1,m_2^2,m_3^2)}\sqrt{\lambda(\sigma_3,m_1^2,m_2^2)}}\right) \\
\zeta^3_{1(2)} &= \operatorname{acos}\left(\frac{2m_3^2(m_1^2+m_2^2-\sigma_3)+(-m_1^2-m_3^2+\sigma_2)(-m_2^2-m_3^2+\sigma_1)}{\sqrt{\lambda(\sigma_1,m_2^2,m_3^2)}\sqrt{\lambda(\sigma_2,m_3^2,m_1^2)}}\right)
\end{aligned}
$$

7.3 Phase space sample

7.3.1 Definition

See also:

AmpForm's Kinematics page.

$$
\begin{cases} 1 & \text{for } \phi\left(\sigma_i, \sigma_j\right) \leq 0 \\ \text{NaN} & \text{otherwise} \end{cases}
$$

$$
\phi\left(\sigma_i, \sigma_j\right) = \lambda\left(\lambda\left(\sigma_j, m_j^2, m_0^2\right), \lambda\left(\sigma_k, m_k^2, m_0^2\right), \lambda\left(\sigma_i, m_i^2, m_0^2\right)\right)
$$

$$
\lambda\left(x, y, z\right) = x^2 - 2xy - 2xz + y^2 - 2yz + z^2
$$

$$
\sigma_k = m_0^2 + m_1^2 + m_2^2 + m_3^2 - \sigma_i - \sigma_j
$$

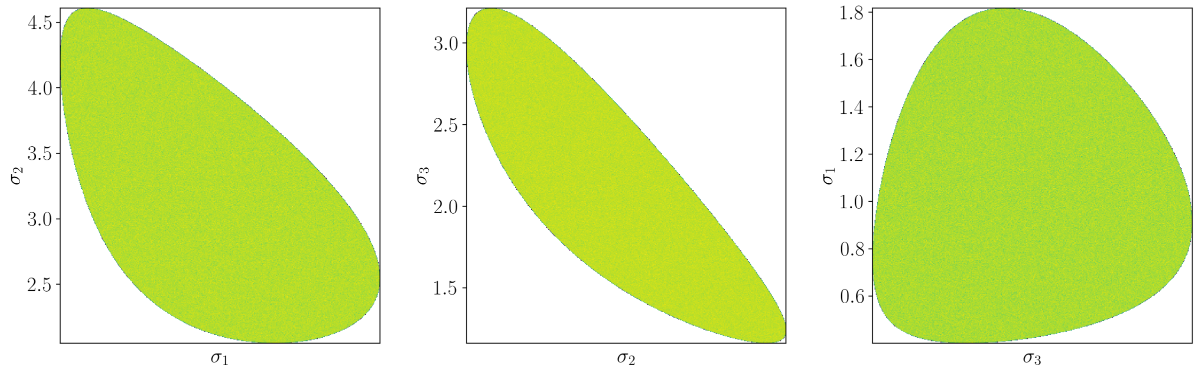
7.3.2 Visualization

$$
\begin{aligned}
m_0 &= 2.28646 \\
m_1 &= 0.938272046 \\
m_2 &= 0.13957018 \\
m_3 &= 0.49367700000000003
\end{aligned}
$$

```
No GPU/TPU found, falling back to CPU. (Set TF_CPP_MIN_LOG_LEVEL=0 and rerun for
more info.)
```

```
<Figure size 500x500 with 1 Axes>
```

```
Generating intensity-based sample:   0%|          | 0/10000000 [00:00<?, ?it/s]
```



7.4 Alignment consistency

$$\sum_{\lambda_0=-1/2}^{1/2}\sum_{\lambda_1=-1/2}^{1/2}\left|\sum_{\lambda_0'=-1/2}^{1/2}\sum_{\lambda_1'=-1/2}^{1/2} A^1_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{1(1)}\right) + A^2_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{2(1)}\right) + A^3_{\lambda\ldots}\right.$$

$$\sum_{\lambda_0=-1/2}^{1/2}\sum_{\lambda_1=-1/2}^{1/2}\left|\sum_{\lambda_0'=-1/2}^{1/2}\sum_{\lambda_1'=-1/2}^{1/2} A^1_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{1(2)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{1(2)}\right) + A^2_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{2(2)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{2(2)}\right) + A^3_{\lambda\ldots}\right.$$

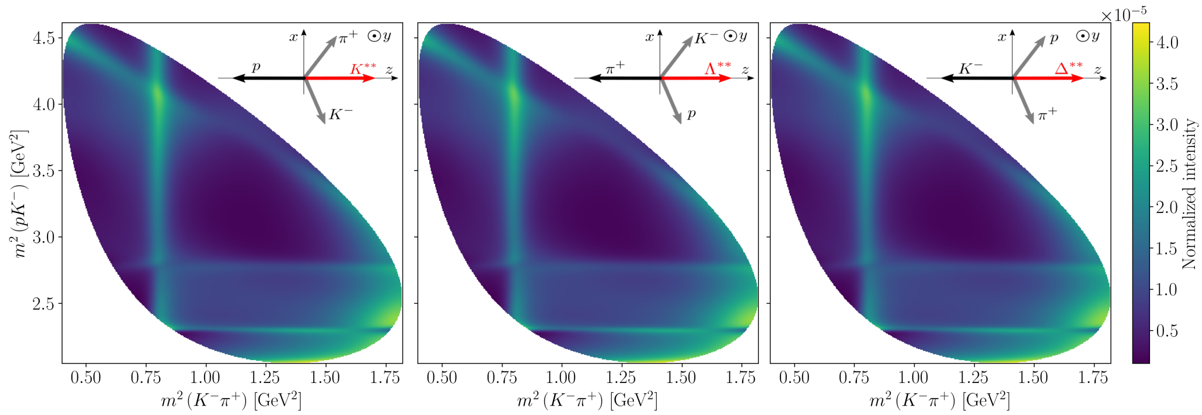
$$\sum_{\lambda_0=-1/2}^{1/2}\sum_{\lambda_1=-1/2}^{1/2}\left|\sum_{\lambda_0'=-1/2}^{1/2}\sum_{\lambda_1'=-1/2}^{1/2} A^1_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{1(3)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{1(3)}\right) + A^2_{\lambda_0',\lambda_1',0,0} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{2(3)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{2(3)}\right) + A^3_{\lambda\ldots}\right.$$

See *DPD angles* (page 40) for the definition of each $\zeta^i_{j(k)}$.

Note that a change in reference sub-system requires the production couplings for certain sub-systems to flip sign:

- **Sub-system 2** as reference system: flip signs of $\mathcal{H}^\text{production}_{K^{**}}$ and $\mathcal{H}^\text{production}_{L^{**}}$
- **Sub-system 3** as reference system: flip signs of $\mathcal{H}^\text{production}_{K^{**}}$ and $\mathcal{H}^\text{production}_{D^{**}}$

```
{1: Array(3.91663029e+08, dtype=float64),
 2: Array(3.91663029e+08, dtype=float64),
 3: Array(3.91663029e+08, dtype=float64)}
```



7.5 Benchmarking

Tip: This notebook benchmarks JAX on a **single CPU core**. Compare with Julia results as reported in ComPWA/polarimetry#27. See also the Extended benchmark #68 discussion.

Note: This notebook uses only one run and one loop for `%timeit`, because JAX seems to cache its return values.

```
Physical cores: 16
Total cores: 16
```

```
CPU times: user 1min 30s, sys: 100 ms, total: 1min 30s
Wall time: 1min 31s
```

7.5.1 `DataTransformer` performance

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
303 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
8.19 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
7.93 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

```
274 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
2.34 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
1.92 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

7.5.2 Parametrized function

Compare *All parameters substituted* (page 45).

Total number of mathematical operations:

- α_x: 133,630
- α_y: 133,634
- α_z: 133,630
- I_{tot}: 43,198

```
CPU times: user 14.2 s, sys: 6.9 ms, total: 14.2 s
Wall time: 14.3 s
```

One data point

JIT-compilation

```
<TimeitResult : 1.08 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 5.62 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Compiled performance

```
<TimeitResult : 420 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 841 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

54x54 grid sample

Compiled but uncached

```
<TimeitResult : 1.19 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 7.08 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 3.84 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 5.69 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

100.000 event phase space sample

Compiled but uncached

```
<TimeitResult : 1.22 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 7.14 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 15.6 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 66.3 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Recompilation after parameter modification

Compiled but uncached

```
<TimeitResult : 1.17 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 7.17 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 14.7 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 78.5 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

7.5.3 All parameters substituted

Compare *Parametrized function* (page 43).

Number of mathematical operations after substituting all parameters:

- α_x: 29,552
- α_y: 29,556
- α_z: 29,552
- I_{tot}: 9,624

```
CPU times: user 5.19 s, sys: 6.93 ms, total: 5.2 s
Wall time: 5.23 s
```

One data point

JIT-compilation

```
<TimeitResult : 689 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 3.58 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Compiled performance

```
<TimeitResult : 181 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 446 µs ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

54x54 grid sample

Compiled but uncached

```
<TimeitResult : 842 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 4.53 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 2.69 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 8.47 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

100.000 event phase space sample

Compiled but uncached

```
<TimeitResult : 891 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 4.75 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

Second run with cache

```
<TimeitResult : 13.9 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

```
<TimeitResult : 67 ms ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)>
```

7.5.4 Summary

```
<pandas.io.formats.style.Styler at 0x7f3a55b83d30>
```

7.6 Serialization

7.6.1 File size checks

File sizes for **100x100 grid**:

File type	Size
`export/alpha-x-arrays.json`	**141 kB**
`export/alpha-x-pandas.json`	**311 kB**
`export/alpha-x-python.json`	**260 kB**
`export/alpha-x-pandas-json.zip`	**51 kB**
`export/alpha-x-pandas.csv`	**129 kB**

7.6.2 Export polarimetry grids

Decided to use the `alpha-x-arrays.json` format. It can be exported with *export_polarimetry_field()* (page 68).

Polarimetry grid can be downloaded here: `export/polarimetry-model-0.json` (540 kB).

7.6.3 Import and interpolate

The arrays in the *exported JSON files* (page 35) can be used to create a `RegularGridInterpolator` for the intensity and for each components of $\vec{\alpha}$.

> *import_polarimetry_field()* (page 68) returns JAX arrays, which are read-only. `RegularGridInterpolator` requires modifiable arrays, so we convert them to NumPy.
>
> Also note that the `values` array needs to be **transposed**!

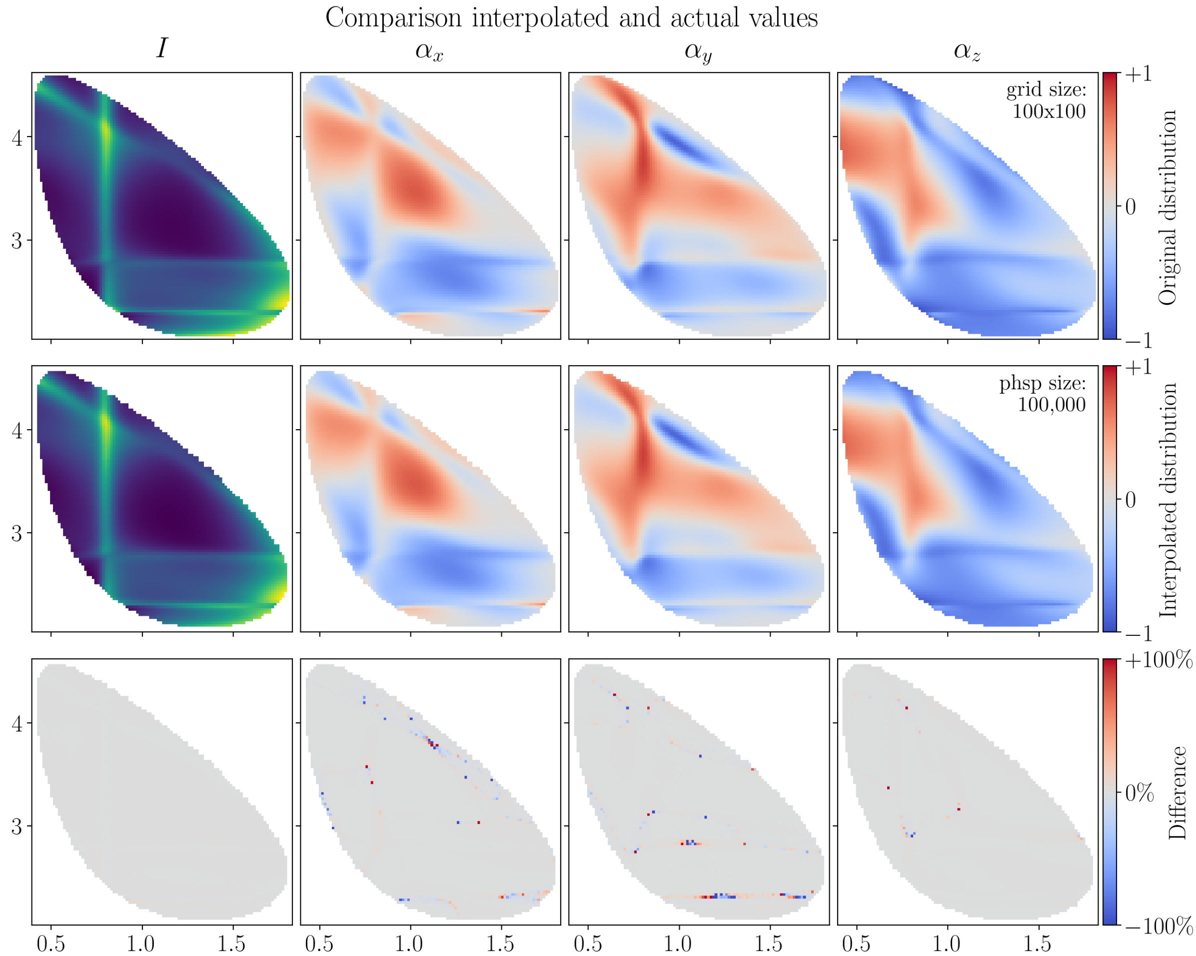
This is a function that can compute an interpolated value of each of these observables for a random point on the Dalitz plane.

```
array([0.18379986])
```

As opposed to SciPy's deprecated `interp2d`, `RegularGridInterpolator` is already in vectorized form, so there is no need to `vectorize` it.

```
Generating intensity-based sample:   0%|          | 0/100000 [00:00<?, ?it/s]
```

```
array([2165.82154945, 5481.04128781, 6254.96174147, ..., 1369.40657535,
       4456.44114915, 7197.97782088])
```



Note: The interpolated values over this phase space sample have been visualized by interpolating again over a `meshgrid` with `scipy.interpolate.griddata`.

Tip: *Determination of polarization* (page 52) shows how this interpolation method can be used to determine the polarization $\vec{P}$ from a given intensity distribution.

7.7 Amplitude model with LS-couplings

7.7.1 Model inspection

$$\sum_{\lambda_0'=-1/2}^{1/2} \sum_{\lambda_1'=-1/2}^{1/2} A^1_{\lambda_0',\lambda_1'} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{1(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{1(1)}\right) + A^2_{\lambda_0',\lambda_1'} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{2(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{2(1)}\right) + A^3_{\lambda_0',\lambda_1'} d^{\frac{1}{2}}_{\lambda_1',\lambda_1}\left(\zeta^1_{3(1)}\right) d^{\frac{1}{2}}_{\lambda_0,\lambda_0'}\left(\zeta^0_{3(1)}\right)$$

$$A^{1}_{-\frac{1}{2},-\frac{1}{2}} = \sum_{\lambda_R=-1}^{1} -\frac{\sqrt{10}\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{3}{2},\lambda_R+\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},\frac{1}{2}}C^{\frac{1}{2},\lambda_R+\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R+\frac{1}{2}}\mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{K(892),0,0}d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} -\frac{\sqrt{2}\delta_{-\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{1}{2},\cdots}_{0,0,\cdots}\cdots}{\cdots}$$

$$A^{2}_{-\frac{1}{2},-\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} -\frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{L(1520),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{L(1520),0,-\frac{1}{2}}d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} -\frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\cdots}_{\frac{3}{2},\cdots}\cdots}{\cdots}$$

$$A^{3}_{-\frac{1}{2},-\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{D(1232),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{D(1232),-\frac{1}{2},0}d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,\cdots}\cdots}{\cdots}$$

$$A^{1}_{-\frac{1}{2},\frac{1}{2}} = \sum_{\lambda_R=-1}^{1} \frac{\sqrt{10}\delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{3}{2},\lambda_R-\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},-\frac{1}{2}}C^{\frac{1}{2},\lambda_R-\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R-\frac{1}{2}}\mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{K(892),0,0}d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} \frac{\sqrt{2}\delta_{-\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{1}{2},\lambda_R-\cdots}_{0,0,\frac{1}{2},\cdots}\cdots}{\cdots}$$

$$A^{2}_{-\frac{1}{2},\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{L(1520),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{L(1520),0,\frac{1}{2}}d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}\cdots}{\cdots}$$

$$A^{3}_{-\frac{1}{2},\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{D(1232),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{D(1232),\frac{1}{2},0}d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{-\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}\cdots}{\cdots}$$

$$A^{1}_{\frac{1}{2},-\frac{1}{2}} = \sum_{\lambda_R=-1}^{1} -\frac{\sqrt{10}\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{3}{2},\lambda_R+\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},\frac{1}{2}}C^{\frac{1}{2},\lambda_R+\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R+\frac{1}{2}}\mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{K(892),0,0}d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} -\frac{\sqrt{2}\delta_{\frac{1}{2},\lambda_R+\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{1}{2},\lambda_R\cdots}_{0,0,\frac{1}{2}\cdots}\cdots}{\cdots}$$

$$A^{2}_{\frac{1}{2},-\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} -\frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{L(1520),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{L(1520),0,-\frac{1}{2}}d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} -\frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,\cdots}\cdots}{\cdots}$$

$$A^{3}_{\frac{1}{2},-\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{D(1232),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{D(1232),-\frac{1}{2},0}d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}\cdots}{\cdots}$$

$$A^{1}_{\frac{1}{2},\frac{1}{2}} = \sum_{\lambda_R=-1}^{1} \frac{\sqrt{10}\delta_{\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{3}{2},\lambda_R-\frac{1}{2}}_{1,\lambda_R,\frac{1}{2},-\frac{1}{2}}C^{\frac{1}{2},\lambda_R-\frac{1}{2}}_{2,0,\frac{3}{2},\lambda_R-\frac{1}{2}}\mathcal{H}^{\mathrm{LS,production}}_{K(892),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{K(892),0,0}d^{1}_{\lambda_R,0}(\theta_{23})}{2} + \sum_{\lambda_R=-1}^{1} \frac{\sqrt{2}\delta_{\frac{1}{2},\lambda_R-\frac{1}{2}}\mathcal{R}(\sigma_1)C^{\frac{1}{2},\lambda_R-\frac{1}{2}}_{0,0,\frac{1}{2},\lambda_R\cdots}\cdots}{\cdots}$$

$$A^{2}_{\frac{1}{2},\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{L(1520),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{L(1520),0,\frac{1}{2}}d^{\frac{3}{2}}_{\lambda_R,-\frac{1}{2}}(\theta_{31})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_2)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C\cdots}{\cdots}$$

$$A^{3}_{\frac{1}{2},\frac{1}{2}} = \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C^{\frac{1}{2},\lambda_R}_{2,0,\frac{3}{2},\lambda_R}\mathcal{H}^{\mathrm{LS,production}}_{D(1232),2,\frac{3}{2}}\mathcal{H}^{\mathrm{decay}}_{D(1232),\frac{1}{2},0}d^{\frac{3}{2}}_{\lambda_R,\frac{1}{2}}(\theta_{12})}{2} + \sum_{\lambda_R=-3/2}^{3/2} \frac{\sqrt{10}\delta_{\frac{1}{2}\lambda_R}\mathcal{R}(\sigma_3)C^{\frac{3}{2},\lambda_R}_{\frac{3}{2},\lambda_R,0,0}C\cdots}{\cdots}$$

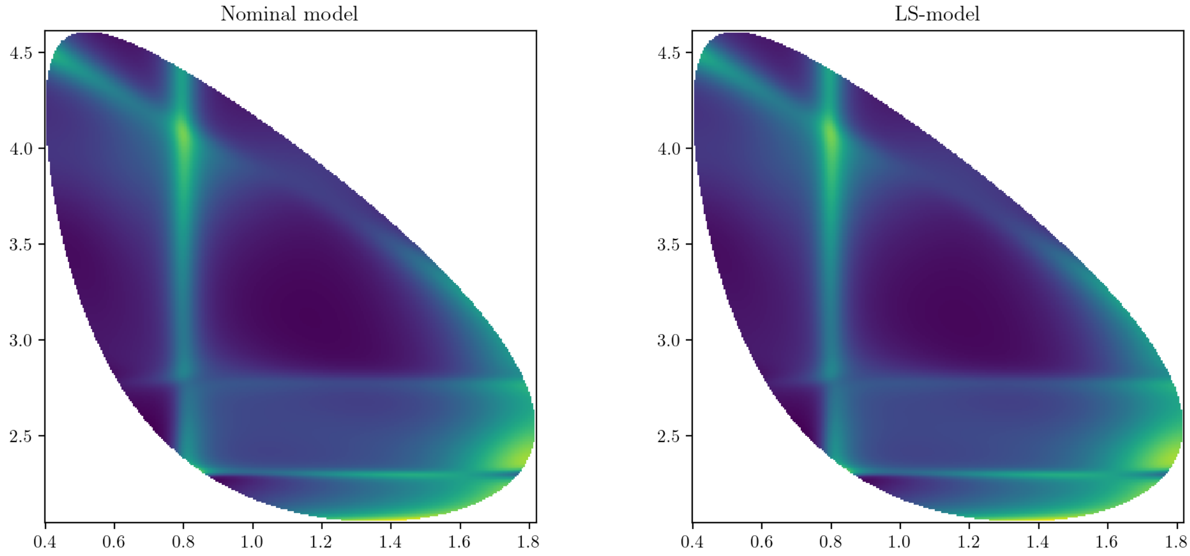
Decay	coupling			factor
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1405),0,\frac{1}{2}}$	$=$	$-1.22-0.0395i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(1405) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1405),1,\frac{1}{2}}$	$=$	$1.81-1.63i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1520),1,\frac{3}{2}}$	$=$	$0.192+0.167i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Lambda(1520) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1520),2,\frac{3}{2}}$	$=$	$-0.116-0.243i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1600),0,\frac{1}{2}}$	$=$	$0.134+0.628i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(1600) \xrightarrow[S=1/2]{L=1} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1600),1,\frac{1}{2}}$	$=$	$1.71-1.13i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1670),0,\frac{1}{2}}$	$=$	$0.0092-0.201i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(1670) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1670),1,\frac{1}{2}}$	$=$	$0.115+0.168i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1690),1,\frac{3}{2}}$	$=$	$-0.379+0.331i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Lambda(1690) \xrightarrow[S=1/2]{L=2} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(1690),2,\frac{3}{2}}$	$=$	$0.286-0.248i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} \Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(2000),0,\frac{1}{2}}$	$=$	$2.81+0.0715i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} \Lambda(2000) \xrightarrow[S=1/2]{L=0} K^-p\pi^+$	$\mathcal{H}^{\text{LS,production}}_{L(2000),1,\frac{1}{2}}$	$=$	$0.891+0.0874i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$	$\mathcal{H}^{\text{LS,production}}_{D(1232),1,\frac{3}{2}}$	$=$	$-1.5+3.16i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Delta(1232) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$	$\mathcal{H}^{\text{LS,production}}_{D(1232),2,\frac{3}{2}}$	$=$	$0.587-0.839i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$	$\mathcal{H}^{\text{LS,production}}_{D(1600),1,\frac{3}{2}}$	$=$	$1.6-2.46i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Delta(1600) \xrightarrow[S=1/2]{L=1} p\pi^+K^-$	$\mathcal{H}^{\text{LS,production}}_{D(1600),2,\frac{3}{2}}$	$=$	$0.432-0.689i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} \Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+K^-$	$\mathcal{H}^{\text{LS,production}}_{D(1700),1,\frac{3}{2}}$	$=$	$-3.16+2.29i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} \Delta(1700) \xrightarrow[S=1/2]{L=2} p\pi^+K^-$	$\mathcal{H}^{\text{LS,production}}_{D(1700),2,\frac{3}{2}}$	$=$	$0.179-0.299i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(700) \xrightarrow[S=0]{L=0} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(700),0,\frac{1}{2}}$	$=$	$-0.000167-0.685i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} K(700) \xrightarrow[S=0]{L=0} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(700),1,\frac{1}{2}}$	$=$	$-0.631+0.0404i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(892),0,\frac{1}{2}}$	$=$	1.0	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(892),1,\frac{1}{2}}$	$=$	$-0.342+0.064i$	-1
$\Lambda_c^+ \xrightarrow[S=3/2]{L=1} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(892),1,\frac{3}{2}}$	$=$	$-0.755-0.592i$	$+1$
$\Lambda_c^+ \xrightarrow[S=3/2]{L=2} K(892) \xrightarrow[S=0]{L=1} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(892),2,\frac{3}{2}}$	$=$	$-0.0938-0.38i$	-1
$\Lambda_c^+ \xrightarrow[S=1/2]{L=0} K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(1430),0,\frac{1}{2}}$	$=$	$-1.35-3.15i$	$+1$
$\Lambda_c^+ \xrightarrow[S=1/2]{L=1} K(1430) \xrightarrow[S=0]{L=0} \pi^+K^-p$	$\mathcal{H}^{\text{LS,production}}_{K(1430),1,\frac{1}{2}}$	$=$	$0.598-0.956i$	$+1$

It is asserted that these amplitude expressions to not evaluate to 0 once the Clebsch-Gordan coefficients are evaluated.

See also:

See *Resonances and LS-scheme* (page 3) for the allowed LS-values.

7.7.2 Distribution



7.7.3 Decay rates

Resonance	Nominal	LS-model	Difference
$\Lambda(1405)$	7.78	7.02	−0.75
$\Lambda(1520)$	1.91	1.95	+0.03
$\Lambda(1600)$	5.16	5.21	+0.05
$\Lambda(1670)$	1.15	1.18	+0.02
$\Lambda(1690)$	1.16	1.09	−0.08
$\Lambda(2000)$	9.55	9.84	+0.30
$\Delta(1232)$	28.73	28.97	+0.24
$\Delta(1600)$	4.50	4.24	−0.26
$\Delta(1700)$	3.89	3.99	+0.10
$K(700)$	2.99	3.25	+0.26
$K(892)$	21.95	21.25	−0.70
$K(1430)$	14.70	15.41	+0.71

Tip: Compare with the values with uncertainties as reported in *Decay rates* (page 32).

7.8 SU(2) → SO(3) homomorphism

The Cornwell theorem from the group theory (see for example Section 3, Chapter 5 of [3]) gives the relation between the rotation of the transition amplitude and the physical vector of polarization sensitivity:

$$R_{ij}(\phi,\theta,\chi) = \frac{1}{2}\mathrm{tr}\left(D^{1/2*}(\phi,\theta,\chi)\sigma_i^P D^{1/2*\dagger}(\phi,\theta,\chi)\sigma_j^P\right)\,, \tag{7.1}$$

where tr represents the trace operation applied to the product of the two-dimensional matrices, D and σ^P, and $R_{ij}(\phi,\theta,\chi)$ is a three-dimensional rotation matrix implementing the Euler transformation to a physical vector.

$$\begin{bmatrix} -\sin(\chi)\sin(\phi)+\cos(\chi)\cos(\phi)\cos(\theta) & -\sin(\chi)\cos(\phi)\cos(\theta)-\sin(\phi)\cos(\chi) & \sin(\theta)\cos(\phi) \\ \sin(\chi)\cos(\phi)+\sin(\phi)\cos(\chi)\cos(\theta) & -\sin(\chi)\sin(\phi)\cos(\theta)+\cos(\chi)\cos(\phi) & \sin(\phi)\sin(\theta) \\ -\sin(\theta)\cos(\chi) & \sin(\chi)\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\begin{bmatrix} -\sin(\chi)\sin(\phi)+\cos(\chi)\cos(\phi)\cos(\theta) & -\sin(\chi)\cos(\phi)\cos(\theta)-\sin(\phi)\cos(\chi) & \sin(\theta)\cos(\phi) \\ \sin(\chi)\cos(\phi)+\sin(\phi)\cos(\chi)\cos(\theta) & -\sin(\chi)\sin(\phi)\cos(\theta)+\cos(\chi)\cos(\phi) & \sin(\phi)\sin(\theta) \\ -\sin(\theta)\cos(\chi) & \sin(\chi)\sin(\theta) & \cos(\theta) \end{bmatrix}$$

7.9 Determination of polarization

Given the aligned polarimeter field $\vec{\alpha}$ and the corresponding intensity distribution I_0, the intensity distribution I for a polarized decay can be computed as follows:

$$I(\phi,\theta,\chi;\tau) = I_0(\tau)\left(1+\vec{P}R(\phi,\theta,\chi)\vec{\alpha}(\tau)\right) \tag{7.2}$$

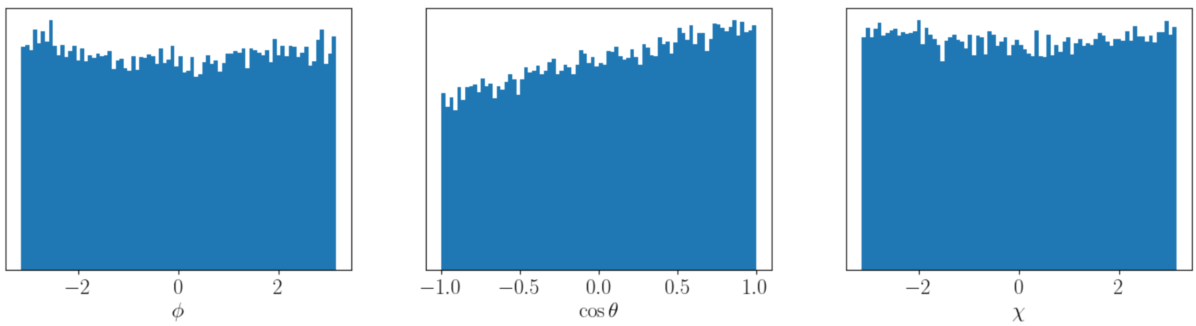
with R the rotation matrix over the decay plane orientation, represented in Euler angles (ϕ,θ,χ).

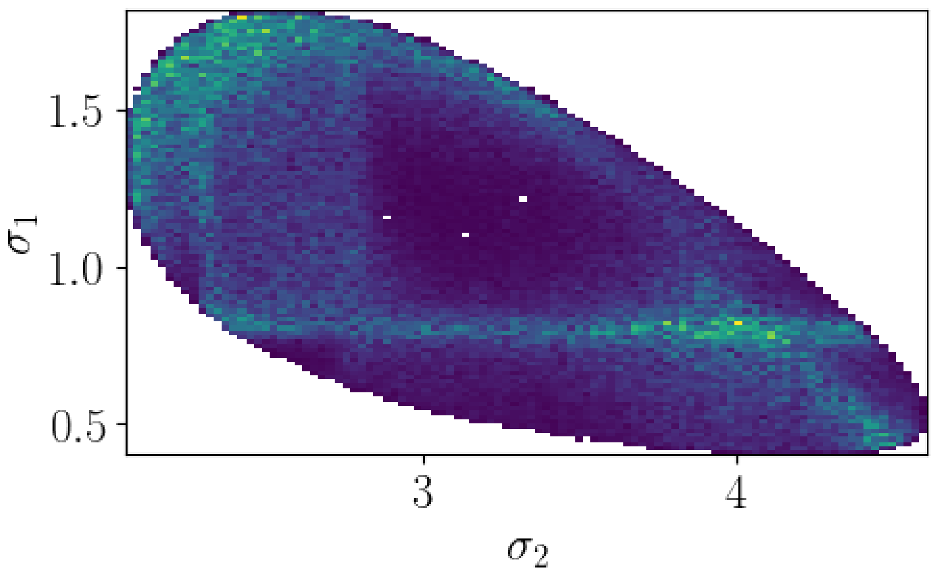
In this section, we show that it's possible to determine the polarization $\vec{P}$ from a given intensity distribution I of a λ_c decay if we the $\vec{\alpha}$ fields and the corresponding I_0 values of that Λ_c decay. We get $\vec{\alpha}$ and I_0 by interpolating the grid samples provided from *Exported distributions* (page 35) using the method described in *Import and interpolate* (page 47). We perform the same procedure with the averaged aligned polarimeter vector from Section 5.6 in order to quantify the loss in precision when integrating over the Dalitz plane variables τ.

7.9.1 Polarized test distribution

For this study, a phase space sample is uniformly generated over the Dalitz plane variables τ. The phase space sample is extended with uniform distributions over the decay plane angles (ϕ,θ,χ), so that the phase space can be used to generate a hit-and-miss toy sample for a polarized intensity distribution.

We now generate an intensity distribution over the phase space sample given a certain value for $\vec{P}$ [1] using Eq. (7.2) and by interpolating the $\vec{\alpha}$ and I_0 fields with the grid samples for the nominal model.





7.9.2 Using the exported polarimeter grid

The generated distribution is now assumed to be a *measured distribution* I with unknown polarization $\vec{P}$. It is shown below that the actual $\vec{P}$ with which the distribution was generated can be found by performing a fit on Eq. (7.2). This is done with `iminuit`, starting with a certain 'guessed' value for $\vec{P}$ as initial parameters.

To avoid having to generate a hit-and-miss intensity test distribution, the parameters $\vec{P} = \left(P_x, P_y, P_z\right)$ are optimized with regard to a **weighted negative log likelihood estimator**:

$$\mathrm{NLL} = -\sum_i w_i \log I_{i,\vec{P}}\left(\phi, \theta, \chi; \tau\right) . \tag{7.3}$$

with the normalized intensities of the generated distribution taken as weights:

$$w_i = n\, I_i \Big/ \sum_j^n I_j \,, \tag{7.4}$$

such that $\sum w_i = n$. To propagate uncertainties, a fit is performed using the exported grids of each alternative model.

```
┌─────────────────────────────────────────────────────────────────────────┐
│                                Migrad                                   │
├──────────────────────────────────┬──────────────────────────────────────┤
│ FCN = 1.127e+06                  │              Nfcn = 66               │
│ EDM = 2.58e-06 (Goal: 0.0001)    │            time = 3.2 sec            │
├──────────────────────────────────┼──────────────────────────────────────┤
│          Valid Minimum           │   Below EDM threshold (goal x 10)    │
├──────────────────────────────────┼──────────────────────────────────────┤
│      No parameters at limit      │           Below call limit           │
├──────────────────────────────────┼──────────────────────────────────────┤
│             Hesse ok             │         Covariance accurate          │
└──────────────────────────────────┴──────────────────────────────────────┘
┌───┬──────┬───────────┬───────────┬────────────┬────────────┬─────────┬─────────┬␣
↪───────┐
│   │ Name │   Value   │ Hesse Err │ Minos Err- │ Minos Err+ │ Limit-  │ Limit+  │␣
↪Fixed │
├───┼──────┼───────────┼───────────┼────────────┼────────────┼─────────┼─────────┼␣
↪───────┤
│ 0 │ Px   │   0.217   │   0.008   │            │            │         │         │␣
↪       │
│ 1 │ Py   │   0.011   │   0.008   │            │            │         │         │␣
↪       │
│ 2 │ Pz   │  -0.665   │   0.007   │            │            │         │         │␣
```

(continues on next page)

(continued from previous page)

```
↪    │
└────┴─────────┴───────────┴───────────┴────────────┴────────────┴─────────┴─────────┴───────┘

┌────┬─────────────────────────────┐
│    │       Px       Py       Pz  │
├────┼─────────────────────────────┤
│ Px │ 6.24e-05        0        0  │
│ Py │        0 6.27e-05        0  │
│ Pz │        0        0  5.6e-05  │
└────┴─────────────────────────────┘
```

The polarization $\vec{P}$ is determined to be (in %):

$$
\begin{aligned}
P_x &= +21.65^{+0.30}_{-0.62} \\
P_y &= +1.08^{+0.02}_{-0.05} \\
P_z &= -66.50^{+1.66}_{-0.85}
\end{aligned}
$$

with the upper and lower sign being the systematic extrema uncertainties as determined by the alternative models.

This is to be compared with the model uncertainties reported by [1]:

$$
\begin{aligned}
P_x &= +21.65 \pm 0.36 \\
P_y &= +1.08 \pm 0.09 \\
P_z &= -66.5 \pm 1.1.
\end{aligned}
$$

The polarimeter values for each model are (in %):

Model	P_x	P_y	P_z	ΔP_x	ΔP_y	ΔP_z
0	$+21.65$	$+1.08$	-66.5			
1	$+21.59$	$+1.07$	-66.4	-0.06	-0.01	$+0.13$
2	$+21.63$	$+1.07$	-66.5	-0.02	-0.00	$+0.04$
3	$+21.69$	$+1.07$	-66.6	$+0.04$	-0.01	-0.10
4	$+21.65$	$+1.10$	-66.5	$+0.00$	$+0.02$	-0.04
5	$+21.68$	$+1.08$	-66.5	$+0.03$	$+0.01$	-0.04
6	$+21.51$	$+1.06$	-66.0	-0.14	-0.02	$+0.48$
7	$+21.18$	$+1.05$	-65.3	-0.47	-0.03	$+1.18$
8	$+21.34$	$+1.03$	-65.6	-0.31	-0.05	$+0.87$
9	$+21.34$	$+1.05$	-65.6	-0.31	-0.03	$+0.90$
10	$+21.95$	$+1.10$	-67.4	$+0.30$	$+0.02$	-0.85
11	$+21.61$	$+1.08$	-66.4	-0.04	$+0.00$	$+0.12$
12	$+21.70$	$+1.03$	-66.6	$+0.05$	-0.05	-0.10
13	$+21.67$	$+1.08$	-66.6	$+0.02$	$+0.00$	-0.05
14	$+21.66$	$+1.08$	-66.5	$+0.01$	$+0.00$	-0.02
15	$+21.03$	$+1.10$	-64.8	-0.62	$+0.02$	$+1.66$
16	$+21.64$	$+1.08$	-66.5	-0.01	$+0.00$	$+0.03$
17	$+21.67$	$+1.08$	-66.6	$+0.02$	$+0.00$	-0.09

7.9.3 Using the averaged polarimeter vector

Equation (7.2) requires knowledge about the aligned polarimeter field $\vec{\alpha}(\tau)$ and intensity distribution $I_0(\tau)$ over all kinematic variables τ. It is, however, also possible to compute the differential decay rate from the averaged polarimeter vector $\vec{\overline{\alpha}}$ (see *Average polarimetry values* (page 33)). The equivalent formula to Eq. (7.2) is:

$$
\frac{8\pi^2}{\Gamma}\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\phi\,\mathrm{d}\cos\theta\,\mathrm{d}\chi} = 1 + \sum_{i,j} P_i R_{ij}(\phi,\theta,\chi)\overline{\alpha}_j\,, \tag{7.5}
$$

We use this equation along with Eq. (7.3) to determine $\vec{P}$ with `Minuit`.

```
┌─────────────────────────────────────────────────────────────────────────┐
│                                Migrad                                   │
├──────────────────────────────────┬──────────────────────────────────────┤
│ FCN = 1.151e+06                  │              Nfcn = 56               │
│ EDM = 6.08e-08 (Goal: 0.0001)    │            time = 2.3 sec            │
├──────────────────────────────────┼──────────────────────────────────────┤
│          Valid Minimum           │   Below EDM threshold (goal x 10)    │
├──────────────────────────────────┼──────────────────────────────────────┤
│      No parameters at limit      │           Below call limit           │
├──────────────────────────────────┼──────────────────────────────────────┤
│             Hesse ok             │         Covariance accurate          │
└──────────────────────────────────┴──────────────────────────────────────┘
┌───┬──────┬───────────┬───────────┬────────────┬────────────┬─────────┬─────────┬␣
↪Fixed │
├───┼──────┼───────────┼───────────┼────────────┼────────────┼─────────┼─────────┼
│   │ Name │   Value   │ Hesse Err │ Minos Err- │ Minos Err+ │ Limit-  │ Limit+  │␣
├───┼──────┼───────────┼───────────┼────────────┼────────────┼─────────┼─────────┼
│ 0 │ Px   │   0.203   │   0.019   │            │            │         │         │␣
↪      │
│ 1 │ Py   │  -0.003   │   0.019   │            │            │         │         │␣
↪      │
│ 2 │ Pz   │  -0.661   │   0.019   │            │            │         │         │␣
↪      │
└───┴──────┴───────────┴───────────┴────────────┴────────────┴─────────┴─────────┴
┌────┬──────────────────────────────┐
│    │       Px        Py        Pz │
├────┼──────────────────────────────┤
│ Px │ 0.000364        -0         0 │
│ Py │       -0 0.000367        -0 │
│ Pz │        0        -0 0.000362 │
└────┴──────────────────────────────┘
```

Using the averaged polarimeter vector $\vec{\overline{\alpha}}$, the polarization $\vec{P}$ is determined to be (in %):

$$
\begin{array}{rcl}
P_x &=& +20.32_{-2.44}^{+1.04} \\
P_y &=& -0.26_{-0.08}^{+0.17} \\
P_z &=& -66.14_{-3.32}^{+7.91}
\end{array}.
$$

The polarimeter values for each model are (in %):

Model	$\mathbf{P_x}$	$\mathbf{P_y}$	$\mathbf{P_z}$	$\mathbf{\Delta P_x}$	$\mathbf{\Delta P_y}$	$\mathbf{\Delta P_z}$
0	$+20.32$	-0.26	-66.1			
1	$+20.23$	-0.24	-65.9	-0.08	$+0.01$	$+0.26$
2	$+20.28$	-0.26	-66.0	-0.04	-0.00	$+0.12$
3	$+20.49$	-0.22	-66.8	$+0.18$	$+0.04$	-0.63
4	$+20.29$	-0.32	-65.9	-0.03	-0.06	$+0.21$
5	$+20.25$	-0.33	-65.8	-0.07	-0.07	$+0.36$
6	$+19.97$	-0.31	-64.9	-0.35	-0.05	$+1.24$
7	$+18.34$	-0.31	-59.7	-1.98	-0.05	$+6.43$
8	$+19.90$	-0.18	-65.0	-0.42	$+0.08$	$+1.17$
9	$+19.46$	-0.25	-63.2	-0.85	$+0.01$	$+2.90$
10	$+21.36$	-0.23	-69.5	$+1.04$	$+0.03$	-3.32
11	$+20.25$	-0.28	-65.9	-0.07	-0.02	$+0.26$
12	$+19.82$	-0.34	-64.2	-0.49	-0.08	$+1.97$
13	$+20.38$	-0.25	-66.3	$+0.06$	$+0.01$	-0.20
14	$+20.35$	-0.25	-66.3	$+0.04$	$+0.00$	-0.12
15	$+17.88$	-0.09	-58.2	-2.44	$+0.17$	$+7.91$
16	$+20.32$	-0.25	-66.1	$+0.00$	$+0.01$	-0.00
17	$+20.29$	-0.22	-66.2	-0.03	$+0.04$	-0.08

Propagating extrema uncertainties

In Section 5.6, the averaged aligned polarimeter vectors with systematic model uncertainties were found to be:

observable	**central**	**stat + syst**
$\overline{\alpha}_x$ $[10^{-3}]$	-62.6	14.8
$\overline{\alpha}_y$ $[10^{-3}]$	$+8.9$	12.7
$\overline{\alpha}_z$ $[10^{-3}]$	-278.0	40.4
$\lvert\overline{\alpha}\rvert$ $[10^{-3}]$	285.1	37.9
$\theta(\overline{\alpha})$ $[\pi]$	$+0.929$	0.017
$\phi(\overline{\alpha})$ $[\pi]$	$+0.955$	0.067

This list of uncertainties is determined by the *extreme deviations* of the alternative models, whereas the uncertainties on the polarizations determined in Section 7.9.3 are determined by the averaged polarimeters of *all* alternative models. The tables below shows that there is a loss in systematic uncertainty when we propagate uncertainties by taking computing $\vec{P}$ *only* with combinations of $\alpha_i - \sigma_i, \alpha_i + \sigma_i$ for each $i \in x, y, z$.

```
0%|          | 0/8 [00:00<?, ?it/s]
```

```
0%|          | 0/8 [00:00<?, ?it/s]
```

Polarizations from $\overline{\alpha}$ in cartesian coordinates:

$$\begin{array}{rcrcr} P_x &=& +20.32 &\pm& 3.60 \\ P_y &=& -0.26 &\pm& 0.34 \\ P_z &=& -66.14 &\pm& 11.51 \end{array}$$

Polarizations from $\overline{\alpha}$ in polar coordinates:

$$\begin{array}{rcrcr} P_x &=& +20.32 &\pm& 3.23 \\ P_y &=& -0.26 &\pm& 0.19 \\ P_z &=& -66.14 &\pm& 10.08 \end{array}$$

α_x	α_y	α_z	P_x	P_y	P_z	ΔP_x	ΔP_y	ΔP_z
-62.6	8.9	-278.0	$+20.32$	-0.26	-66.14			
-77.4	-3.8	-318.4	$+17.7$	-0.25	-57.4	-2.58	$+0.01$	$+8.7$
-77.4	-3.8	-237.5	$+23.3$	-0.55	-74.9	$+2.97$	-0.30	-8.7
-77.4	$+21.6$	-318.4	$+17.6$	-0.28	-57.4	-2.72	-0.02	$+8.7$
-77.4	$+21.6$	-237.5	$+23.0$	-0.60	-74.7	$+2.71$	-0.34	-8.6
-47.8	-3.8	-318.4	$+17.9$	-0.04	-58.4	-2.43	$+0.21$	$+7.8$
-47.8	-3.8	-237.5	$+23.9$	-0.21	-77.7	$+3.60$	$+0.05$	-11.5
-47.8	$+21.6$	-318.4	$+17.7$	-0.07	-58.3	-2.57	$+0.19$	$+7.8$
-47.8	$+21.6$	-237.5	$+23.6$	-0.26	-77.5	$+3.31$	$+0.00$	-11.3

$\lvert\alpha\rvert$	θ $[\pi]$	ϕ $[\pi]$	P_x	P_y	P_z	ΔP_x	ΔP_y	ΔP_z
285.1	0.929	0.955	$+20.32$	-0.26	-66.14			
247.1	$+0.91$	$+0.89$	$+23.3$	-0.45	-76.1	$+3.01$	-0.19	-10.0
247.1	$+0.91$	$+1.02$	$+23.5$	-0.44	-75.9	$+3.23$	-0.19	-9.8
247.1	$+0.95$	$+0.89$	$+23.2$	-0.12	-76.2	$+2.91$	$+0.14$	-10.1
247.1	$+0.95$	$+1.02$	$+23.4$	-0.12	-76.1	$+3.05$	$+0.14$	-10.0
323.0	$+0.91$	$+0.89$	$+17.9$	-0.35	-58.2	-2.47	-0.09	$+7.9$
323.0	$+0.91$	$+1.02$	$+18.0$	-0.34	-58.1	-2.30	-0.08	$+8.0$
323.0	$+0.95$	$+0.89$	$+17.8$	-0.09	-58.3	-2.54	$+0.17$	$+7.8$
323.0	$+0.95$	$+1.02$	$+17.9$	-0.09	-58.2	-2.44	$+0.17$	$+7.9$

7.9.4 Increase in uncertainties

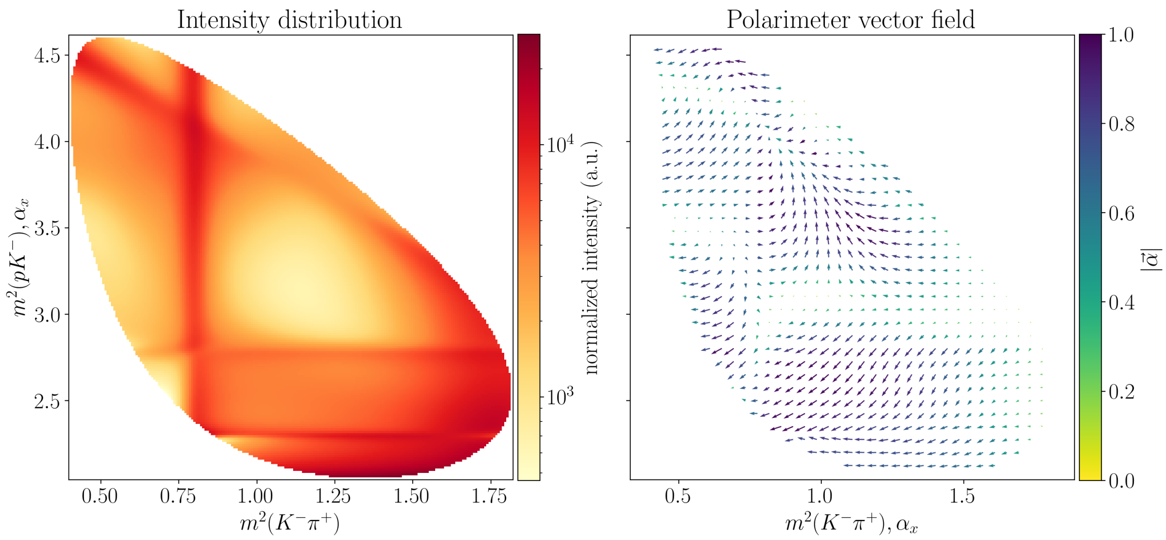
When the polarization is determined with the averaged aligned polarimeter vector $\vec{\overline{\alpha}}$ instead of the aligned polarimeter vector field $\vec{\alpha}(\tau)$ over all Dalitz variables τ, the uncertainty is expected to increase by a factor $S_0/\overline{S}_0 \approx 3$, with:

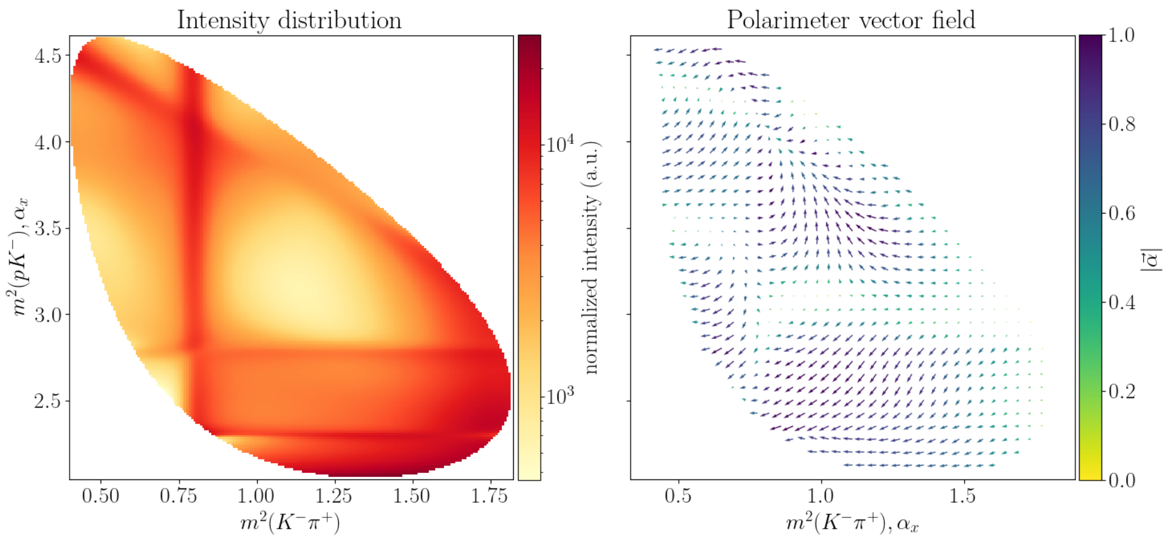
$$\begin{aligned} S_0^2 &= 3 \int I_0 \left|\vec{\alpha}\right|^2 \mathrm{d}^n\tau \Big/ \int I_0 \, \mathrm{d}^n\tau \\ \overline{S}_0^2 &= 3(\overline{\alpha}_x^2 + \overline{\alpha}_y^2 + \overline{\alpha}_z^2)\,. \end{aligned} \tag{7.6}$$

The following table shows the maximal deviation (systematic uncertainty) of the determined polarization $\vec{P}$ for each alternative model (determined with the $\overline{\alpha}$-values in cartesian coordinates). The second and third column indicate the systematic uncertainty (in %) as determined with the full vector field and with the averaged vector, respectively.

σ_{model}	$\vec{\alpha}(\tau)$	$\vec{\overline{\alpha}}$	factor
P_x	0.62	2.44	3.9
P_y	0.05	0.17	3.5
P_z	1.66	7.91	4.8

7.10 Interactive visualization





Tip: Run this notebook locally in Jupyter or online on Binder to modify parameters interactively!

CHAPTER

EIGHT

BIBLIOGRAPHY

CHAPTER

NINE

POLARIMETRY

```
import polarimetry
```

formulate_polarimetry(*builder: DalitzPlotDecompositionBuilder (page 61), reference_subsystem: Literal[1, 2, 3] = 1*) → tuple[PoolSum, PoolSum, PoolSum]

Submodules and Subpackages

9.1 amplitude

```
import polarimetry.amplitude
```

class AmplitudeModel(*decay: ThreeBodyDecay (page 65), intensity: Expr = 1, amplitudes: dict[Indexed, Expr] = _Nothing.NOTHING, variables: dict[Symbol, Expr] = _Nothing.NOTHING, parameter_defaults: dict[Symbol, float] = _Nothing.NOTHING*)

Bases: `object`

decay: *ThreeBodyDecay* **(page 65)**

intensity: Expr

amplitudes: dict[Indexed, Expr]

variables: dict[Symbol, Expr]

parameter_defaults: dict[Symbol, float]

property full_expression: Expr

class DalitzPlotDecompositionBuilder(*decay: ThreeBodyDecay (page 65), min_ls: bool = True*)

Bases: `object`

formulate(*reference_subsystem: Literal[1, 2, 3] = 1, cleanup_summations: bool = False*) → *AmplitudeModel* (page 61)

formulate_subsystem_amplitude(*λ0: Rational, λ1: Rational, λ2: Rational, λ3: Rational, subsystem_id: Literal[1, 2, 3]*) → *AmplitudeModel* (page 61)

formulate_aligned_amplitude(*λ0: Rational | Symbol, λ1: Rational | Symbol, λ2: Rational | Symbol, λ3: Rational | Symbol, reference_subsystem: Literal[1, 2, 3] = 1*) → tuple[PoolSum, dict[Symbol, Expr]]

get_indexed_base(*typ: Literal['production', 'decay'], min_ls: bool = True*) → IndexedBase

Get a basis to generate coupling symbols for the production or decay node.

class DynamicsConfigurator(*decay: ThreeBodyDecay (page 65)*)

Bases: `object`

register_builder(*identifier, builder: DynamicsBuilder (page 62)*) → None

get_builder(*identifier*) → *DynamicsBuilder* (page 62)

property decay: *ThreeBodyDecay* (page 65)

class DynamicsBuilder(**args, **kwargs*)

Bases: `Protocol`

simplify_latex_rendering() → None

Improve LaTeX rendering of an `Indexed` object.

Submodules and Subpackages

9.1.1 angles

```
import polarimetry.amplitude.angles
```

formulate_scattering_angle(*state_id: int, sibling_id: int*) → tuple[Symbol, acos]

Formulate the scattering angle in the rest frame of the resonance.

Compute the θ_{ij} scattering angle as formulated in Eq (A1) in the DPD paper. The angle is that between particle i and spectator particle k in the rest frame of the isobar resonance (ij).

formulate_theta_hat_angle(*isobar_id: int, aligned_subsystem: int*) → tuple[Symbol, acos]

Formulate an expression for $\hat{\theta}_{i(j)}$.

formulate_zeta_angle(*rotated_state: int, aligned_subsystem: int, reference_subsystem: int*) → tuple[Symbol, acos]

Formulate an expression for the alignment angle $\zeta^i_{j(k)}$.

9.2 lhcb

```
import polarimetry.lhcb
```

Import functions that are specifically for this LHCb analysis.

See also:

Cross-check with LHCb data (page 13)

load_model(*model_file: Path | str, particle_definitions: dict[str, Particle (page 65)], model_id: int | str = 0*) → *AmplitudeModel* (page 61)

load_model_builder(*model_file: Path | str, particle_definitions: dict[str, Particle (page 65)], model_id: int | str = 0*) → *DalitzPlotDecompositionBuilder* (page 61)

load_three_body_decay(*resonance_names: Iterable[str], particle_definitions: dict[str, Particle (page 65)], min_ls: bool = True*) → *ThreeBodyDecay* (page 65)

class ParameterBootstrap(*filename: Path | str, decay: ThreeBodyDecay (page 65), model_id: int | str = 0*)

Bases: `object`

A wrapper for loading parameters from `model-definitions.yaml`.

property values: dict[str, complex | float | int]

property uncertainties: dict[str, complex | float | int]

create_distribution(*sample_size: int, seed: int | None = None*) → dict[str, complex | float | int]

load_model_parameters(*filename: Path | str, decay: ThreeBodyDecay (page 65), model_id: int | str = 0, particle_definitions: dict[str, Particle (page 65)] | None = None*) → dict[Indexed | Symbol, complex | float]

load_model_parameters_with_uncertainties(*filename: Path | str, decay: ThreeBodyDecay (page 65), model_id: int | str = 0, particle_definitions: dict[str, Particle (page 65)] | None = None*) → dict[Indexed | Symbol, *MeasuredParameter* (page 63)]

flip_production_coupling_signs(*obj: _T, subsystem_names: Iterable[Pattern]*) → _T

compute_decay_couplings(*decay: ThreeBodyDecay (page 65)*) → dict[Indexed, *MeasuredParameter* (page 63)[int]]

ParameterType

Template for the parameter type of a for `MeasuredParameter` (page 63).

alias of TypeVar('ParameterType', complex, float)

class MeasuredParameter(*value: ParameterType (page 63), hesse: ParameterType (page 63), model: ParameterType (page 63) | None = None, systematic: ParameterType (page 63) | None = None*)

Bases: `Generic`[`ParameterType` (page 63)]

Data structure for imported parameter values.

`MeasuredParameter.value` (page 63) and `hesse` (page 63) are taken from the supplemental material, whereas `model` (page 63) and `systematic` (page 63) are taken from Tables 8 and 9 from the original LHCb paper [1].

value: *ParameterType* (page 63)

Central value of the parameter as determined by a fit with Minuit.

hesse: *ParameterType* (page 63)

Parameter uncertainty as determined by a fit with Minuit.

model: *ParameterType* (page 63) | None

Systematic uncertainties from fit bootstrapping.

systematic: *ParameterType* (page 63) | None

Systematic uncertainties from detector effects etc..

property uncertainty: *ParameterType* (page 63)

get_conversion_factor(*resonance: Particle (page 65)*) → Literal[-1, 1]

get_conversion_factor_ls(*resonance: Particle (page 65), L: Rational, S: Rational*) → Literal[-1, 1]

parameter_key_to_symbol(*key: str, min_ls: bool = True, particle_definitions: dict[str, Particle (page 65)] | None = None*) → Indexed | Symbol

extract_particle_definitions(*decay: ThreeBodyDecay (page 65)*) → dict[str, *Particle* (page 65)]

Submodules and Subpackages

9.2.1 dynamics

```
import polarimetry.lhcb.dynamics
```

formulate_bugg_breit_wigner(*decay_chain: ThreeBodyDecayChain (page 66)*) → tuple[*BuggBreitWigner* (page 66), dict[Symbol, float]]

formulate_exponential_bugg_breit_wigner(*decay_chain: ThreeBodyDecayChain (page 66)*) → tuple[Mul, dict[Symbol, float]]

See this paper, Eq. (4).

formulate_flatte_1405(*decay: ThreeBodyDecayChain (page 66)*) → tuple[*FlattéSWave* (page 66), dict[Symbol, float]]

formulate_breit_wigner(*decay_chain: ThreeBodyDecayChain (page 66)*) → tuple[*BreitWignerMinL* (page 66), dict[Symbol, float]]

9.2.2 particle

```
import polarimetry.lhcb.particle
```

Hard-coded particle definitions.

load_particles(*filename: Path | str*) → dict[str, *Particle* (page 65)]

Load `Particle` (page 65) definitions from a YAML file.

class ResonanceJSON(**args, **kwargs*)

Bases: `dict`

latex: str

jp: str

mass: float | str

width: float | str

9.3 data

```
import polarimetry.data
```

create_data_transformer(*model: AmplitudeModel (page 61), backend: str = 'jax'*) → SympyDataTransformer

create_phase_space_filter(*decay: ThreeBodyDecay (page 65), x_mandelstam: Literal[1, 2, 3] = 1, y_mandelstam: Literal[1, 2, 3] = 2, outside_value=nan*) → PositionalArgumentFunction

generate_meshgrid_sample(*decay: ThreeBodyDecay (page 65), resolution: int, x_mandelstam: Literal[1, 2, 3] = 1, y_mandelstam: Literal[1, 2, 3] = 2*) → DataSample

Generate a `numpy.meshgrid` sample for plotting with `matplotlib.pyplot`.

generate_sub_meshgrid_sample(*decay: ThreeBodyDecay (page 65), resolution: int, x_range: tuple[float, float], y_range: tuple[float, float], x_mandelstam: Literal[1, 2, 3] = 1, y_mandelstam: Literal[1, 2, 3] = 2*) → DataSample

generate_phasespace_sample(*decay: ThreeBodyDecay (page 65), n_events: int, seed: int | None = None*) → DataSample

Generate a uniform distribution over Dalitz variables $\sigma_{1,2,3}$.

compute_dalitz_boundaries(*decay: ThreeBodyDecay (page 65)*) → tuple[tuple[float, float], tuple[float, float], tuple[float, float]]

create_mass_symbol_mapping(*decay: ThreeBodyDecay (page 65)*) → dict[Symbol, float]

9.4 decay

```
import polarimetry.decay
```

Data structures that describe a three-body decay.

class Particle(*name: str, latex: str, spin: SupportsFloat, parity: Literal[-1, 1], mass: float, width: float*)

Bases: `object`

name: str

latex: str

spin: Rational

parity: Literal[(-1, 1)]

mass: float

width: float

class IsobarNode(*parent: Particle (page 65), child1: Particle (page 65) | IsobarNode (page 65), child2: Particle (page 65) | IsobarNode (page 65), interaction: LSCoupling (page 66) | tuple[int, SupportsFloat] | None = None*)

Bases: `object`

parent: *Particle* (page 65)

child1: *Particle* (page 65) | *IsobarNode* (page 65)

child2: *Particle* (page 65) | *IsobarNode* (page 65)

interaction: *LSCoupling* (page 66) | None

property children: tuple[*Particle* (page 65), *Particle* (page 65)]

class ThreeBodyDecay(*states: OuterStates (page 66), chains: tuple[ThreeBodyDecayChain (page 66), ...]*)

Bases: `object`

states: *OuterStates* (page 66)

chains: tuple[*ThreeBodyDecayChain* (page 66), ...]

property initial_state: *Particle* (page 65)

property final_state: dict[Literal[1, 2, 3], *Particle* (page 65)]

find_chain(*resonance_name: str*) → *ThreeBodyDecayChain* (page 66)

get_subsystem (*subsystem_id: Literal[1, 2, 3]*) → *ThreeBodyDecay* (page 65)

get_decay_product_ids (*spectator_id: Literal[1, 2, 3]*) → tuple[int, int]

OuterStates

Mapping of the initial and final state IDs to their `Particle` (page 65) definition.

alias of `Dict`[`Literal`[0, 1, 2, 3], `Particle` (page 65)]

class ThreeBodyDecayChain (*decay: IsobarNode (page 65)*)

Bases: `object`

decay: *IsobarNode* (page 65)

property parent: *Particle* (page 65)

property resonance: *Particle* (page 65)

property decay_products: tuple[*Particle* (page 65), *Particle* (page 65)]

property spectator: *Particle* (page 65)

property incoming_ls: *LSCoupling* (page 66)

property outgoing_ls: *LSCoupling* (page 66)

class LSCoupling (*L: int, S: SupportsFloat*)

Bases: `object`

L: int

S: Rational

9.5 dynamics

```
import polarimetry.dynamics
```

Functions for dynamics lineshapes and kinematics.

See also:

Dynamics lineshapes (page 39)

class P (*s, mi, mj, **hints*)

Bases: `UnevaluatedExpression`

class Q (*s, m0, mk, **hints*)

Bases: `UnevaluatedExpression`

class BreitWignerMinL (*s, decaying_mass, spectator_mass, resonance_mass, resonance_width, child2_mass, child1_mass, l_dec, l_prod, R_dec, R_prod*)

Bases: `UnevaluatedExpression`

class BuggBreitWigner (*s, m0, Γ0, m1, m2, γ*)

Bases: `UnevaluatedExpression`

class FlattéSWave (*s, m0, widths, masses1, masses2*)

Bases: `UnevaluatedExpression`

class EnergyDependentWidth (*s, m0, Γ0, m1, m2, L, R*)

Bases: `UnevaluatedExpression`

class BlattWeisskopf (*z, L, **hints*)

Bases: `UnevaluatedExpression`

9.6 function

```
import polarimetry.function
```

compute_sub_function (*func: ParametrizedFunction*, *input_data: DataSample*, *non_zero_couplings: list[Pattern]*)

set_parameter_to_zero (*func: ParametrizedFunction*, *search_term: Pattern*) → None

interference_intensity (*func*, *data*, *chain1: list[str]*, *chain2: list[str]*) → float

sub_intensity (*func*, *data*, *non_zero_couplings: list[str]*)

integrate_intensity (*intensities*) → float

9.7 io

```
import polarimetry.io
```

Input-output functions for `ampform` and `sympy` objects.

Functions in this module are registered with `functools.singledispatch()` and can be extended as follows:

```
>>> from polarimetry.io import as_latex
>>> @as_latex.register(int)
... def _(obj: int) -> str:
...     return "my custom rendering"
>>> as_latex(1)
'my custom rendering'
>>> as_latex(3.4 - 2j)
'3.4-2i'
```

This code originates from ComPWA/ampform#280.

as_latex (*obj*, ***kwargs*) → str

as_latex (*obj: complex*, ***kwargs*) → str

as_latex (*obj: Basic*, ***kwargs*) → str

as_latex (*obj: Mapping*, ***kwargs*) → str

as_latex (*obj: Iterable*, ***kwargs*) → str

as_latex (*obj:* IsobarNode *(page 65)*, ***kwargs*) → str

as_latex (*obj:* ThreeBodyDecay *(page 65)*, ***kwargs*) → str

as_latex (*obj:* ThreeBodyDecayChain *(page 66)*, ***kwargs*) → str

as_latex (*obj:* Particle *(page 65)*, *with_jp: bool = False*, *only_jp: bool = False*, ***kwargs*) → str

Render objects as a LaTeX `str`.

The resulting `str` can for instance be given to `IPython.core.display.Math`.

Optional keywords:

- only_jp: Render a `Particle` (page 65) as J^P value (spin-parity) only.
- with_jp: Render a `Particle` (page 65) with value J^P value.

as_markdown_table (*obj: Sequence*) → str

Render objects a `str` suitable for generating a table.

display_latex (*obj*) → None

display_doit (*expr: UnevaluatedExpression*, *deep=False*, *terms_per_line: int | None = None*) → None

perform_cached_doit (*unevaluated_expr: Expr*, *directory: str | None = None*) → Expr

Perform `doit()` on an `Expr` and cache the result to disk.

The cached result is fetched from disk if the hash of the original expression is the same as the hash embedded in the filename.

Parameters

- **unevaluated_expr** – A `sympy.Expr` on which to call `doit()`.
- **directory** – The directory in which to cache the result. If `None`, the cache directory will be put under the home directory, or to the path specified by the environment variable `SYMPY_CACHE_DIR`.

Tip: For a faster cache, set PYTHONHASHSEED to a fixed value.

See also:

perform_cached_lambdify() (page 68)

perform_cached_lambdify (*expr: Expr*, *parameters: Mapping[Symbol, ParameterValue] | None = None*, *backend: str = 'jax'*, *directory: str | None = None*) → ParametrizedFunction | Function

Lambdify a SymPy `Expr` and cache the result to disk.

The cached result is fetched from disk if the hash of the expression is the same as the hash embedded in the filename.

Parameters

- **expr** – A `sympy.Expr` on which to call `create_function()` or `create_parametrized_function()`.
- **parameters** – Specify this argument in order to create a `ParametrizedBackendFunction` instead of a `PositionalArgumentFunction`.
- **backend** – The choice of backend for the created numerical function. **WARNING**: this function has only been tested for `backend="jax"`!
- **directory** – The directory in which to cache the result. If `None`, the cache directory will be put under the home directory, or to the path specified by the environment variable `SYMPY_CACHE_DIR`.

Tip: For a faster cache, set PYTHONHASHSEED to a fixed value.

See also:

perform_cached_doit() (page 68)

get_readable_hash (*obj*) → str

mute_jax_warnings () → None

export_polarimetry_field (*sigma1: Array*, *sigma2: Array*, *alpha_x: Array*, *alpha_y: Array*, *alpha_z: Array*, *intensity: Array*, *filename: str*, *metadata: dict | None = None*) → None

import_polarimetry_field (*filename: str*, *steps: int = 1*) → dict[str, Array]

9.8 plot

```
import polarimetry.plot
```

Helper functions for `matplotlib`.

add_watermark(*ax: Axes*, *x: float = 0.03*, *y: float = 0.03*, *fontsize: int | None = None*, ***kwargs*) → None

get_contour_line(*contour_set: QuadContourSet*) → LineCollection

use_mpl_latex_fonts(*reset_mpl: bool = True*) → None

stylize_contour(*contour_set: QuadContourSet*, ***, *edgecolor=None*, *label: str | None = None*, *linestyle: str | None = None*, *linewidth: float | None = None*) → None

9.9 spin

```
import polarimetry.spin
```

generate_ls_couplings(*parent_spin: SupportsFloat*, *child1_spin: SupportsFloat*, *child2_spin: SupportsFloat*, *max_L: int = 3*) → list[tuple[int, Rational]]

```
>>> generate_ls_couplings(1.5, 0.5, 0)
[(1, 1/2), (2, 1/2)]
```

filter_parity_violating_ls(*ls_couplings: list[tuple[int, Rational]]*, *parent_parity: SupportsInt*, *child1_parity: SupportsInt*, *child2_parity: SupportsInt*) → list[tuple[int, Rational]]

```
>>> LS = generate_ls_couplings(0.5, 1.5, 0)  # Λc → Λ(1520)π
>>> LS
[(1, 3/2), (2, 3/2)]
>>> filter_parity_violating_ls(LS, +1, -1, -1)
[(2, 3/2)]
```

create_spin_range(*spin: SupportsFloat*) → list[Rational]

```
>>> create_spin_range(1.5)
[-3/2, -1/2, 1/2, 3/2]
```

create_rational_range(*__from: SupportsFloat*, *__to: SupportsFloat*) → list[Rational]

```
>>> create_rational_range(-0.5, +1.5)
[-1/2, 1/2, 3/2]
```

Notebook execution times

Document	Modified	Method	Run Time (s)	Status
amplitude-model (page 3)	2023-08-24 22:03	cache	26.46	✅
appendix/alignment (page 42)	2023-08-24 22:05	cache	83.63	✅
appendix/angles (page 40)	2023-08-24 22:05	cache	2.77	✅
appendix/benchmark (page 43)	2023-08-24 22:08	cache	175.67	✅
appendix/dynamics (page 39)	2023-08-24 22:08	cache	2.41	✅
appendix/homomorphism (page 51)	2023-08-24 22:08	cache	3.2	✅
appendix/ls-model (page 48)	2023-08-24 22:11	cache	153.17	✅
appendix/phase-space (page 41)	2023-08-24 22:11	cache	24.01	✅
appendix/serialization (page 46)	2023-08-24 22:12	cache	68.9	✅
appendix/widget (page 57)	2023-08-24 22:22	cache	594.92	✅
cross-check (page 13)	2023-08-24 22:23	cache	72.96	✅
intensity (page 21)	2023-08-24 22:28	cache	281.26	✅
polarimetry (page 23)	2023-08-24 22:32	cache	247.35	✅
resonance-polarimetry (page 37)	2023-08-24 23:27	cache	3319.87	✅
uncertainties (page 27)	2023-08-24 23:50	cache	1372.82	✅
zz.polarization-fit (page 52)	2023-08-24 23:55	cache	256.2	✅

BIBLIOGRAPHY

[1] LHCb Collaboration *et al*. Amplitude analysis of $\Lambda_c^+ \to pK^-\pi^+$ decays from semileptonic production. *Phys. Rev. D*, 2022. doi:10.48550/arXiv.2208.03262.

[2] M. Mikhasenko *et al*. Dalitz-plot decomposition for three-body decays. *Phys. Rev. D*, 101(3):034033, February 2020. doi:10.1103/PhysRevD.101.034033.

[3] J. F. Cornwell. *Group Theory in Physics: An Introduction*. Academic Press, San Diego, CA, 1997. ISBN:978-0-12-189800-7.

PYTHON MODULE INDEX

p

INDEX

A

B

C

D

E

F

Q

R

S

T

U

V

W